Dynamic Modeling of Linear Object Deformation Considering Contact with Obstacles

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Abstract-This paper describes the dynamic modeling of linear object deformation considering geometrical constraints and contact with obstacles. Deformable linear objects such as cables and strings are widely used in our daily life, some industries, and medical operations. Modeling, control, and manipulation of deformable linear objects are keys to many applications. We have formulated the static deformation of a linear object using the differential geometry coordinates. In this paper, we apply differential geometry coordinates to the dynamic modeling of linear objects. First, we formulate dynamic 2D deformation of an inextensible linear object based on a differential geometry coordinate system. Second, we consider dynamic deformation of the linear object when forces/moments and geometrical constraints are imposed on the object. Third, we model contact of a linear object with a circular obstacle. It can be applied to self-contact of the linear object. Finally, we show simulation results using the proposed modeling technique.

Keywords—linear object, deformation, dynamics, modeling, contact

I. INTRODUCTION

Deformable linear objects such as cables and strings are widely used in our daily life, some industries, and medical operations. Modeling, control, and manipulation of deformable linear objects are keys to many applications: robotic systems capable of manipulating linear objects, automatic handling of electric cables and wires, and simulation of medical surgery with threads. For example, Fig.1 shows an example of twisted fancy yarns in apparel industries. Such yarn is produced by twisting more than three threads together changing feed speed of each thread intermittently. As the shape of the fancy yarn is dependent on the feed speed, simulation of twisting process is required for design of the fancy yarn. In such simulation, dynamic effect of a thread can not be negligible. Furthermore, contact between a thread and a twisting machine and/or another thread also must be considered.

Deformed shape of threads in a fabric has been described geometrically [1]. In computer graphics, the particle-based approach has been applied to simulate the motion of hairs. Flexure and extension of hairs have been described in [2], while flexure and torsion of hairs have been described in [3], implying that flexure, torsion, and extension of a linear object can be described using particle-based approach. Shinichi Hirai Dept. Robotics, Ritsumeikan Univ. Kusatsu, Shiga 525-8577, Japan Email: hirai@se.ritsumei.ac.jp



Fig. 1. Example of twisted fancy yarns

Deformation of a linear object can be modeled using beam elements in FEM. Spline-based modeling has been applied to the realtime simulation of soft tissues as well as sutures in surgery [4]. Linear objects have been approximated using beams in the engineering community; models exist to describe small deflection of beams[5], and also large deformation using nonlinear beam finite elements[6]. Fast algorithms have been introduced to describe linear object deformation using the Cosserat formulation [7]. In robotics, to eliminate vibration of a linear object during manipulation, FEM has been applied to modeling of its dynamics[8]. Dynamic modeling of a flexible object with an arbitrary shape has been proposed to manipulate it without vibration[9]. We have proposed the differential geometry coordinates to describe the 2D/3D deformation of a linear object with the minimum number of parameters[10]. We have established static formulation of a deformable linear object using the differential geometry coordinates and we have also applied them to the dynamic modeling of the object[11].

In this paper, we propose the dynamic modeling of linear object deformation considering geometrical constraints and contact with obstacles. First, we formulate dynamic 2D deformation of an inextensible linear object based on a differential geometry coordinate system. Second, we consider dynamic deformation of the linear object when forces/moments and geometrical constraints are imposed on the object. Third, we model contact of a linear object with a circular obstacle. It can be applied to self-contact of the linear object. Finally, we show simulation results using the proposed modeling technique.

II. DYNAMIC MODELING OF LINEAR OBJECT DEFORMATION

In this section, we formulate the 2D deformation of an inextensible linear object. Assume that a linear object of length



Fig. 2. Dynamic 2D deformation of inextensible linear object

L bends in frame O-xy, as illustrated in Fig.2. One end of the object is fixed to but can rotate around the origin. The other end is free to move. Let s be the distance from the fixed end along the linear object. Let P(s) be a point on the object specified by distance s. Let $\theta(s, t)$ be the angle from the horizon at point P(s) and time t. Position of point P(s) at time t is then described by

$$\boldsymbol{x}(s,t) = \begin{bmatrix} x(s,t) \\ y(s,t) \end{bmatrix} = \int_0^s \begin{bmatrix} \cos\theta(u,t) \\ \sin\theta(u,t) \end{bmatrix} du.$$
(1)

Differentiating the above equation with respect to time t yields the velocity vector at point P(s) given by

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \int_0^s \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \dot{\theta} \, du. \tag{2}$$

Lagrangian formulation requires the kinetic energy of the object. Kinetic energy T can be described as follows:

$$T = \int_0^L \frac{1}{2}\rho(\dot{x}^2 + \dot{y}^2) \, ds \tag{3}$$

where ρ denotes the line density at point P(s), which may depend on s.

Let us divide region [0, L] into n small regions with a constant interval h = L/n. Let P₀ through P_n be nodal points on the region, of which coordinates are denoted as s_0 through s_n . Applying piecewise linear approximation to function $\theta(s, t)$, the function can be described by

$$\theta(s,t) = \theta_i(t)N_{i,j}(s) + \theta_j(t)N_{j,i}(s)$$
(4)

in a divided region $[s_i, s_j]$, where $\theta_i(t) = \theta(s_i, t)$ and $\theta_j(t) = \theta(s_j, t)$. Shape function $N_{i,j}(s)$ takes 1 at s_i and 0 at s_j while $N_{j,i}(s)$ takes 0 at s_i and 1 at s_j . Let A_i be a set of nodal points adjacent to nodal point P_i :

$$A_i = \{ j \mid \text{nodal point } P_j \text{ is adjacent to nodal point } P_i \}.$$

Then, the shape of the object is represented as links of circular arcs. This representation allows to approximate the object shape with less nodal points than a representation using links of straight segments.

Finite element approximation is applied to the kinetic energy T of a deformable linear object. First, let us introduce the following integrals:

$$S_{i,j}(s;\theta_0,\cdots,\theta_n) = \int_0^s \{\sin\theta(u,t)\} N_{i,j}(u) \ du,$$
$$C_{i,j}(s;\theta_0,\cdots,\theta_n) = \int_0^s \{\cos\theta(u,t)\} N_{i,j}(u) \ du.$$

These integrals $S_{i,j}$ and $C_{i,j}$ depend on θ_0 through θ_n . Then, kinetic energy T is described by a quadratic form with respect to $\dot{\theta}_0$ through $\dot{\theta}_n$ given by

$$T = \frac{1}{2} \sum_{i} \sum_{k} m_{i,k} \dot{\theta}_{i} \dot{\theta}_{k}$$
(5)

where

$$m_{i,k} = \int_0^L \sum_{j \in A_i} \sum_{l \in A_k} \rho(S_{i,j}S_{k,l} + C_{i,j}C_{k,l}) \, ds.$$
(6)

Let M be a matrix of which the (i, k)-th element is given by $m_{i,k}$. Matrix M is referred to as the inertial matrix, which is symmetric and positive-definite.

Partial derivative of inertia matrix component $m_{i,k}$ with respect to a generalized coordinate θ_r is given by

$$\frac{\partial m_{i,k}}{\partial \theta_r} = \int_0^L \sum_{j \in A_i} \sum_{l \in A_k} \rho \left(\frac{\partial S_{i,j}}{\partial \theta_r} S_{k,l} + S_{i,j} \frac{\partial S_{k,l}}{\partial \theta_r} + \frac{\partial C_{i,j}}{\partial \theta_r} C_{k,l} + C_{i,j} \frac{\partial C_{k,l}}{\partial \theta_r} \right) ds$$
(7)

where

$$\frac{\partial S_{i,j}}{\partial \theta_r} = \begin{cases} \int_0^s \cos \theta \cdot N_{i,j}^2 \, du & r = i \\ \int_0^s \cos \theta \cdot N_{i,j} \cdot N_{j,i} \, du & r = j \\ 0 & \text{otherwise} \end{cases}$$
$$\frac{\partial C_{i,j}}{\partial \theta_r} = \begin{cases} \int_0^s -\sin \theta \cdot N_{i,j}^2 \, du & r = i \\ \int_0^s -\sin \theta \cdot N_{i,j} \cdot N_{j,i} \, du & r = j \\ 0 & \text{otherwise} \end{cases}$$

Lagrange equation of motion with respect to θ_i is formulated as follows:

$$\frac{\partial L}{\partial \theta_i} - \frac{d}{ds} \frac{\partial L}{\partial \dot{\theta}_i} = 0.$$
(8)

Contribution of kinetic energy T to the above Lagrange equation of motion is described by

$$\sum_{k} X_{i,k} \dot{\theta}_k - \sum_{k} m_{i,k} \ddot{\theta}_k + \sum_{j} Y_{i,j} \dot{\theta}_j \tag{9}$$

where

$$X_{i,k} = \frac{1}{2} \left\{ \sum_{j} \left(\frac{\partial m_{k,j}}{\partial \theta_i} - \frac{\partial m_{i,j}}{\partial \theta_k} \right) \dot{\theta}_j \right\}, \quad (10)$$

$$Y_{i,j} = \frac{1}{2} \left\{ -\sum_{k} \frac{\partial m_{i,j}}{\partial \theta_k} \dot{\theta}_k \right\}.$$
 (11)

Let θ be a vector consisting of θ_0 through θ_n . Contribution of kinetic energy T to a set of Lagrange equations of motion with respect to θ_0 through θ_n is then summarized in a vector form as follows:

$$X\dot{\theta} - M\ddot{\theta} + Y\dot{\theta}$$

Assume that the potential energy of the object U consists of flexural potential energy U_f and gravitational potential energy U_q , namely,

$$U = U_f + U_g. \tag{12}$$

Assuming that the bending moment at point P(s) on the object is proportional to the curvature at that point, the flexural potential energy is formulated as

$$U_f = \int_0^L \frac{1}{2} R_f \left(\frac{d\theta}{ds}\right)^2 ds \tag{13}$$

where R_f denotes the flexural rigidity at point P(s). Assuming that rigidity R_f is constant, flexural potential energy is approximated as follows:

$$U_f = \sum_{[s_i, s_j]} \frac{1}{2} \begin{bmatrix} \theta_i & \theta_j \end{bmatrix} \frac{R_f}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix}.$$
 (14)

Namely, flexural potential energy is described by a quadratic form as follows:

$$U_f = \frac{1}{2} \boldsymbol{\theta}^{\mathrm{T}} K \boldsymbol{\theta} \tag{15}$$

where K denotes the stiffness matrix of flexural deformation. Consequently, contribution of flexural potential energy to a set of Lagrange equations of motion is given by $-K\theta$. The gravitational potential energy is given by

$$U_g = \int_0^L \rho g \ y(s,t) \ ds \tag{16}$$

where g is the acceleration of gravity. Let $-G(\theta)$ be the contribution of gravitational potential energy to a set of Lagrange equations of motion. The *i*-th component of vector G coincides with the partial derivative of U_g with respect to θ_i :

$$G_i = \frac{\partial U_g}{\partial \theta_i} = \int_0^L \rho g \frac{\partial y}{\partial \theta_i} ds \tag{17}$$

where

$$\frac{\partial y}{\partial \theta_i} = \int_0^s \cos \theta(u, t) \left\{ \sum_{j \in A_i} N_{i,j}(u) \right\} du.$$

Applying any numerical integration, we can compute the contribution of gravitational potential energy to a set of Lagrange equations of motion. As a result, contribution of potential energy U to a set of Lagrange equations of motion with respect to θ_0 through θ_n is summarized in a vector form as follows:

$$-K\boldsymbol{\theta} - \boldsymbol{G}(\boldsymbol{\theta}).$$

A set of Lagrange equations of motion with respect to $\dot{\theta}_0$ through $\dot{\theta}_n$ is then described by

$$X\dot{\boldsymbol{\theta}} - M\ddot{\boldsymbol{\theta}} + Y\dot{\boldsymbol{\theta}} - K\boldsymbol{\theta} - \boldsymbol{G}(\boldsymbol{\theta}) = \boldsymbol{0}.$$
 (18)

Let ω be a vector consisting of angular velocities θ_0 through θ_n . The above set of equations can be rewritten as follows:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega},$$

 $M\dot{\boldsymbol{\omega}} = X\boldsymbol{\omega} + Y\boldsymbol{\omega} - K\boldsymbol{\theta} - \boldsymbol{G}(\boldsymbol{\theta}).$ (19)

Note that matrices M, X, and Y depend on vector θ . Individual elements of matrix M can be computed by eq.(6). Individual elements of matrices X and Y can be calculated through partial derivatives given in eq.(7). We can compute these partial derivatives using any numerical integration method.

In this section, we have formulated the 2D dynamic deformation of an inextensible linear object, which is described by one function $\theta(s, t)$. This formulation can be extended to the 2D dynamic deformation of an extensible linear object, which is described by two independent functions.

III. FORMULATION OF CONSTRAINTS

In this section, we formulate dynamic deformation of a linear object when forces/moments and geometrical constraints are imposed on it.

First, let us assume that force f is imposed on a linear object at point P_a specified by distance s_a . Then, work W done by force f is given by

$$W = \boldsymbol{f} \cdot \boldsymbol{x}(s_a). \tag{20}$$

When work is considered, Lagrangian is described by

$$L = T - U + W. \tag{21}$$

Let $F(\theta)$ be the contribution of work to a set of Lagrange equations of motion. The *i*-th component of vector F coincides with the partial derivative of W with respect to θ_i :

$$F_{i} = \frac{\partial W}{\partial \theta_{i}} = \mathbf{f} \cdot \int_{0}^{s_{a}} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \cdot \left\{ \sum_{j \in A_{i}} N_{i,j}(u) \right\} du.$$
(22)

Note that F_i is equal to zero when i > j where $(j - 1)h < s_a < jh$. Then, Lagrange equations of motion are as follows:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega},$$

 $M\dot{\boldsymbol{\omega}} = X\boldsymbol{\omega} + Y\boldsymbol{\omega} - K\boldsymbol{\theta} - \boldsymbol{G}(\boldsymbol{\theta}) + \boldsymbol{F}(\boldsymbol{\theta}).$ (23)

We can also derive Lagrange equations of motion when a moment is imposed on the object.

Next, let us assume that geometrical constraints are imposed on a linear object. Let $R_j(\theta) = 0$ be geometrical constraints with respect to θ . When constraints exist, Lagrangian is described by

$$L = T - U + \sum_{j} \lambda_j R_j \tag{24}$$

where λ_j are Lagrange multipliers. In this paper, we introduce Constraint Stabilization Method[12] to derive dynamic deformation of a linear object satisfying geometrical constraints. Let us introduce the following equations:

$$\ddot{R}_j + 2\mu_j \dot{R}_j + \mu_j^2 R_j = 0$$
(25)

where μ_j are predetermined values. These differential equations correspond to those of a critical damping with respect to R_j . This implies that R_j converges to zero quickly, namely, constraint $R_j = 0$ is satisfied during the computation. Adding these differential equations to a set of equations of motion, θ and ω satisfying these constraints can be computed. For example, let us assume that the angle at the left endpoint of a linear object is fixed to horizontal angle, *i.e.*,

$$\theta_0 = 0. \tag{26}$$

An additional differential equation is then described by

$$\dot{\omega}_0 = -2\mu\omega_0 - \mu^2\theta_0. \tag{27}$$

Let $\alpha = [1, 0, \dots, 0]^{T}$ be a (n+1)-dimensional vector. Then, we have the following equations:

$$\begin{bmatrix} M & \boldsymbol{\alpha} \\ \boldsymbol{\alpha}^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} X\boldsymbol{\omega} + Y\boldsymbol{\omega} - K\boldsymbol{\theta} - \boldsymbol{G}(\boldsymbol{\theta}) \\ -2\mu\omega_0 - \mu^2\theta_0 \end{bmatrix}.$$
(28)

Solving the above equations, dynamic deformation of a linear object rotation around the left endpoint of which is fixed is derived. Thus, we can also simulate dynamic deformation of a linear object with forces/moments and geometrical constraints.

IV. MODELING OF CONTACT

In this section, we explain modeling of contact between a linear object and a circular obstacle. Let $\boldsymbol{x}_o = [x_o, y_o]^T$ be coordinates of the center of a circular obstacle with radius r. Let P_c be the closest point on a linear object to the center of the obstacle specified by distance s_c . Let us define the distance from the center of the obstacle to P_c as the following vector \boldsymbol{d} :

$$\boldsymbol{d} = \boldsymbol{x}(s_c) - \boldsymbol{x}_o. \tag{29}$$

Let u be the unit vector of the distance d, namely, u = d/|d|. We represent interaction between a linear object and an obstacle as Voigt model of viscoelastic deformation. Let us assume that the following repulsive force f_c of which direction corresponds to the normal to the obstacle surface at P_c is imposed on the object when P_c interferes with the obstacle, *i.e.*, |d| - r < 0:

$$\boldsymbol{f}_c = k_c (r\boldsymbol{u} - \boldsymbol{d}) - b_c (\dot{\boldsymbol{d}} \cdot \boldsymbol{u}) \boldsymbol{u}$$
(30)

where k_c and b_c are spring and damping constants of Voigt model, respectively. Note that k_c and b_c must be determined experimentally. Then, work W done by repulsive force f_c is represented as follows:

$$W = \boldsymbol{f}_c \cdot \boldsymbol{x}_c \tag{31}$$

Substituting eq.(31) into eq.(21), we can simulate dynamic deformation of a linear object contacting with a circular obstacle.

We can also model self-contact of a linear object applying the above technique. Let us assume that a part of a linear object represented as $s \in [s_{s1}, s_{e1}]$ and its another part represented as $s \in [s_{s2}, s_{e2}]$ interfere with each other. The former is denoted as part 1 and the latter as part 2. Let $P_{c1}(s_{c1})$ be the midpoint of part 1, namely, $s_{c1} = (s_{s1} + s_{e1})/2$ and $P_{c2}(s_{c2})$ be the midpoint of part 2, namely, $s_{c2} = (s_{s2} + s_{e2})/2$. Coordinates of midpoints P_{c1} and P_{c2} are

$$\boldsymbol{x}_1 = \begin{bmatrix} x(s_{c1},t) \\ y(s_{c1},t) \end{bmatrix}, \quad \boldsymbol{x}_2 = \begin{bmatrix} x(s_{c2},t) \\ y(s_{c2},t) \end{bmatrix}, \quad (32)$$

respectively. Let d be the following vector:

$$\boldsymbol{d} = \boldsymbol{x}_2 - \boldsymbol{x}_1. \tag{33}$$

We represent interaction between two parts of the object as Voigt model of viscoelastic deformation. Let us assume that the following force f_{c1} is imposed on part 1:

$$\boldsymbol{f}_{c1} = k_c \boldsymbol{d} - b_c (\boldsymbol{d} \cdot \boldsymbol{u}) \boldsymbol{u} \tag{34}$$

where u = d/|d|. At the same time, force $f_{c2} = -f_{c1}$ is imposed on part 2. Consequently, the following work should be added to Lagrangian:

$$W = \boldsymbol{f}_{c1} \cdot \boldsymbol{x}_1 + \boldsymbol{f}_{c2} \cdot \boldsymbol{x}_2. \tag{35}$$

Thus, we can simulate dynamic deformation of a linear object contacting with circular obstacles and itself.

V. SIMULATION RESULTS

This section describes three simulation results of dynamic 2D deformation of an inextensible linear object.

Figure 3 shows a sequence of deformed shapes of a linear object of length 1.00, flexural rigidity 10.00, and line density 10.00 under gravity. The length is divided into 10 small intervals; implying that the deformation can be approximated by 11 nodal points. The distance between the two endpoints of the object is reduced to 0.60 along the horizontal line at its initial shape, as illustrated in Fig.3-(a). The left endpoint of the object is fixed to the coordinate origin. Angles of both endpoint are fixed to horizontal angle, *i.e.*,

$$\theta(0,t) = 0, \quad \theta(L,t) = 0, \quad \forall t \in [0,\infty].$$
(36)

The right endpoint is available to move along x-axis, *i.e.*,

$$y(L,t) = 0, \quad \forall t \in [0,\infty].$$
(37)

The right endpoint is translated at velocity -3.00 from time 0.00 to 0.10, *i.e.*,

$$x(L,t) = \begin{cases} 0.60 - 3.00t, & 0 \le t \le 0.10, \\ 0.30, & 0.10 < t. \end{cases}$$
(38)

That is, four geometrical constraints described by eqs.(36) through (38) are imposed on the object. Runge-Kutta Fehlberg method integrates a set of motion equations of the object. As shown in Fig.3-(b) through (d), motion of the upper part of the object is delayed due to inertia at time 0.03 through 0.09. At time 0.12, the upper part catches up to the lower part as described in Fig.3-(e) and it overruns at time 0.15 through 0.18 as shown in Fig.3-(f) through (g) although the motion of the right endpoint has been stopped. Finally, the object starts to swing as illustrated in Fig.3-(h) through (l). This simulation result given in the figures shows dynamic deformation of a linear object well qualitatively.

Figure 4 shows another sequence of deformed shapes of a linear object of length 1.00, flexural rigidity 0.10, and line density 1.00 with 11 nodal points under gravity. The left endpoint is fixed to the coordinate origin but is free to rotate.



Fig. 3. Simulation of dynamic deformation of linear object with geometrical constraints

At time 0.00, the object is in a horizontal position as shown in Fig.4-(a), *i.e.*,

$$\theta(s,0) = \pi, \quad \forall s \in [0,L]. \tag{39}$$

Coordinates of the center of a circular obstacle are x_{o} = $[-0.30, -0.35]^{T}$ and its radius is 0.01. Spring and damping constants are set to $k_c = 1000$ and $b_c = 100$, respectively. Runge-Kutta Fehlberg method integrates a set of motion equations of the object. First, as illustrated in Fig.4-(b) through (c), the object falls down due to gravity. At time 0.30, the intermediate part of the object collides with the obstacle as plotted in Fig.4-(d). Then, the motion of the intermediate part is immediately stopped due to the repulsive force. Instead, the free end of the object is swung and it twines around the obstacle as shown in Fig.4-(e) through (g). In Fig.4-(g), curvature of the object takes larger. This implies that flexural potential energy is stored in the object. Furthermore, gravitational potential energy also increases. As such potential energy is released, the free end of the object is swung back as described in Fig.4-(h) through (l).

Figure 5 shows the third result of self-contact of a linear object of length 1.00, flexural rigidity 0.02, and line density 0.006 with 11 nodal points under gravity. The distance between the two endpoints of the object is reduced to 0.30 along the horizontal line at its initial shape, as illustrated in Fig.5-(a). The left endpoint of the object is fixed to the coordinate origin and the right endpoint is available to move along x-axis. Angles of both endpoint are fixed to horizontal angle. The right endpoint is translated at velocity -1.50 from time 0.00 to 0.133. Spring and damping constants are set to $k_c = 1000$ and $b_c = 100$, respectively. Runge-Kutta Fehlberg method integrates a set of motion equations of the linear object. As illustrated in Fig.5-(d), the object collides with itself at time 0.12. After collision, the lower parts of the object deform without penetrating each other as shown in Fig.5-(e) through

(h). Due to repulsive forces, the upper part of the object swells as described in Fig.5-(f) through (h).

The above computations were performed on a 600MHz Alpha21164A CPU with 704MB memory operated by DIGITAL UNIX V4.0D. Programs were compiled by DEC C Compiler V5.6 with optimization option -O4. It took about 100 CPU seconds to compute one step of the dynamic deformation in all computations.

Thus, using our proposed technique, we can simulate dynamic deformation of a linear object considering geometrical constraints and contact with other objects including itself.

VI. CONCLUSION

In this paper, the dynamic modeling of linear object deformation considering geometrical constraints and contact with obstacles was proposed. First, we formulated dynamic 2D deformation of an inextensible linear object based on a differential geometry coordinate system. Second, we extended the above formulation to dynamic deformation of the linear object when forces/moments and geometrical constraints are imposed on the object. Third, we modeled contact of a linear object with a circular obstacle. It can be applied to self-contact of the linear object. Finally, we demonstrated simulation results using the proposed modeling technique. They showed dynamic deformation of a linear object well qualitatively.

We will compare simulation results in this paper with experimental results to verify the validity of our proposed technique in future work.

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