# Representation and Planning of Deformable Linear Object Manipulation Including Knotting

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**Abstract**
A qualitative representation method and a rough planning method of linear object manipulation including knotting in the three-dimensional space is proposed. Firstly, states of a linear object are represented as finite permutations of crossing points including the crossing type of each crossing point. Secondly, state transitions among those states are defined. They correspond to operations which change the number of crossing points or permutate their sequence. Then, we can generate possible sequences of crossing state transitions, that is, possible manipulation processes when the initial state and the objective state are given. Thirdly, a method for determination of grasping points and their moving direction in order to realize derived manipulation processes is proposed. Furthermore, some criteria for evaluation of manipulation processes are introduced in order to narrow down candidates for manipulation plans. Finally, that our proposed method can be applied to the rough planning of linear object manipulation is demonstrated.
**Keywords:** Linear Object, Manipulation, Knotting, Planning

## 1. Introduction

The majority of manipulative tasks, including grasping and assembly, are performed through mechanical contact. As rigid object manipulation can be represented as a sequence of finite contact states and their transitions, planning methods for it using contact state graphs have been studied. Hirai et al. had analysed manipulation process of rigid objects. He represented it as a network whose nodes correspond to contact states and whose arcs correspond to state transitions[1]. This method has been applied to the planning of manipulation or assembly[2]. However, systematic approach to the planning of deformable object manipulation has not been established yet. We proposed a qualitative representation method of thin object manipulation considering the contact state of the object and applied it to manipulation planning[3]. Linear objects such as strings and wires can be used for fixing or packing of objects. Such manipulative tasks includes knotting. They can be also knotted in order to make them compact for their storage or transportation. On the other hand, self-entwining of linear objects should be avoided when we manipulate them. When they are both knotted and entwined, they contact with themselves at some points or regions. Therefore, a modeling of manipulation process of linear objects considering self-contact is required for analysis of their manipulative tasks and planning of their manipulation. In this paper, we represent manipulation of a linear object as finite states and their transitions, and apply this representation method to the manipulation planning.

Firstly, a qualitative representation method of the crossing state of a linear object in three-dimensional space is proposed. Secondly, state transitions among those states are defined by introducing four kinds of basic operations. By use of this method, a manipulation process of a linear object can be represented as a sequence of crossing state transitions. Thirdly, a procedure to determine grasping points and their moving direction in order to realize manipulation processes is explained. Furthermore, some criteria for evaluation of manipulation processes in the qualitative analytical phase are introduced. Finally, that our developed system based on this method can be useful for the manipulation planning is demonstrated.

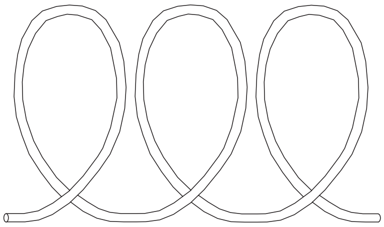
## 2. Representation of Crossing States

Manipulation can be defined as to change the position/direction of an object by contacting controllable objects like manipulators with it and by imposing forces/moments on it. So, we can represent a manipulation process as contact states of a manipulated object with other objects and their transitions. Especially, self-contact of linear objects is important because it occurs when they are knotted or entwined. Moreover, the kind of knots, for example, overhand knot, figure of eight knot, or slipknot, depends on the state of self-contact. However, it seems that contact states of linear objects exist infinitely because they can deform variously. Therefore, we propose a method in order to represent self-contact of linear objects as finite states.

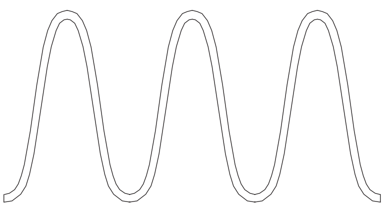
First, let us project the shape of a linear object which deforms in the three-dimensional space on a plane. Then, it appears to contact, in other words, cross with itself at some points on the plane. Note that how the object appears to cross depends on the viewpoint. For example, a spiral linear object appears both to intersect with itself as shown in Fig.1(a) and to have no crossing points as shown in Fig.1(b). In this study, those are identified as different states each other.

Next, let us number crossing points of the object along it. Fig.2 shows an example of a knotted linear object. It has 5 crossing points and their sequence is $E_l$-$C_1$-$C_2$-$C_3$-$C_4$-$C_5$-$C_1$-$C_2$-$C_5$-$C_4$-$C_3$-$E_r$ where $E_l$, $E_r$, and $C_i (i = 1, \cdots, 5)$ represent the left endpoint, the right endpoint, and crossing points, respectively. Thus, we can identify the state of a linear object considering the sequence of its crossing points. Furthermore, we can distinguish two types of crossing: one is to cross so that the front part overlaps from the right side of the rear part as shown in Fig.3(a) and the other is the opposite crossing as shown in Fig.3(b). Let us define the former as the left hand helix crossing $C^+$ and the latter as the right hand helix crossing $C^-$. Then, crossing point sequence of the object shown in Fig.2 becomes $E_l$-$C_1^+$-$C_2^+$-$C_3^-$-$C_4^-$-$C_5^+$-$C_1^+$-$C_2^+$-$C_5^+$-$C_4^-$-$C_3^-$-$E_r$.

Thus, we can represent the state of linear objects, especially knotted ones as finite crossing states.



(a) three intersections



(b) no intersection

Fig. 1 Projection of spiral shape on different planes

### 3. Definition of State Changing Operations

Next, let us consider state transitions among crossing states represented in the previous section. In order to change the crossing state of a linear object, some operations are required. Therefore, a state transition corresponds to an operation which changes the number of crossing points or permutates their sequence. In this study, four kinds of basic operations are prepared as shown in Fig.4. Operation type-I, type-

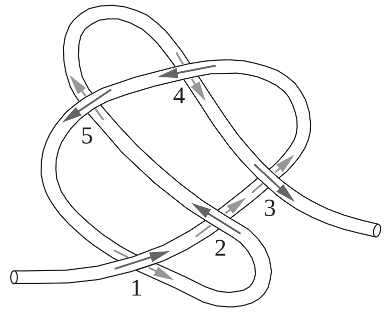
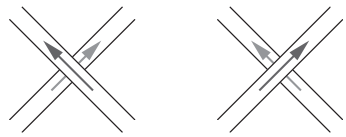



Fig. 2 Example of knotted linear object



(a) (b)

Fig. 3 Kinds of crossing points

II, and type-III are equivalent to Reidemeister move type-I, type-II and type-III in the knot theory[4], respectively. Type-IV operation is needed because a linear object has endpoints in general although the object is a loop and does not have endpoints in the knot theory. By type-I, type-II, and type-IV operation, the number of crossing points is increased or decreased. Type-III operation changes not the number of crossing points but the sequence of them.

Furthermore, let us define an operation to increase crossing points as a crossing operation, an operation to decrease them as a raveling operation, and an operation not to change the number of them as a arranging operation. For example, by type-II raveling operation to crossing points $C_4$ and $C_5$, the crossing state shown in Fig.2 is changed from $E_l$-$C_1^+$-$C_2^+$-$C_3^-$-$C_4^-$-$C_5^+$-$C_1^+$-$C_2^+$-$C_5^+$-$C_4^-$-$C_3^-$-$E_r$ to $E_l$-$C_1^+$-$C_2^+$-$C_3^-$-$C_1^+$-$C_2^+$-$C_3^-$-$E_r$. Note that crossing points is renumbered after a basic operation is executed.

In this study, We describe a raveling operation by using the kind of basic operations and the number of crossing points deleted by it: $\mathrm{RO}_{type}(C_1, C_2, \cdots C_n)$. In the previous example, the operation can be represented as $\mathrm{RO}_{\mathrm{II}}(C_4, C_5)$.

The number of possible crossing operations with respect to one crossing state can be very larger than that of possible raveling operations. Therefore, when the initial state in which a object has no crossing points and the objective state in which it is knotted are given, we search for sequences of raveling operations by which the crossing state is changed from the objective one to the initial one at first. Next, by following found sequences backward, possible sequences of crossing state transitions, that is, qualitative manipulation processes can be derived.

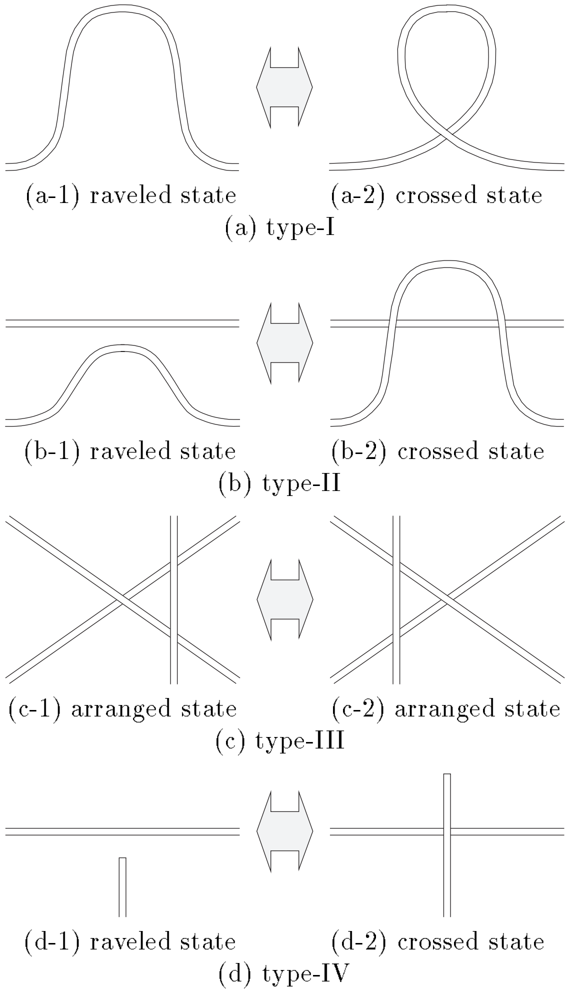



Fig. 4 State changing operation

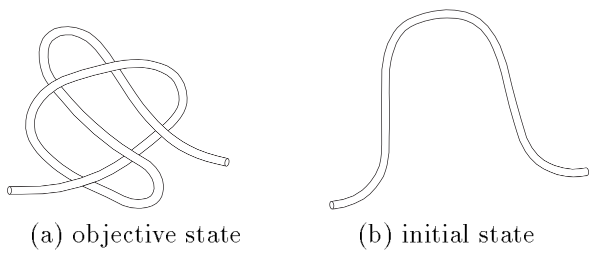



Fig. 5 Example of required manipulation

Fig.5 shows an example of a required manipulation. The objective state in Fig.5(a) can be represented as $E_l$-$C_1^+$-$C_2^+$-$C_3^-$-$C_4^-$-$C_5^+$-$C_1^+$-$C_2^+$-$C_5^+$-$C_4^-$-$C_3^-$-$E_r$ and the initial state in Fig.5(b) can be represented as $E_l$-$E_r$. When we assume that only raveling operations can be used, that is, without arranging operations (type-III operations), 14 crossing states and 32 state transitions as shown in Fig.6 are derived. For example, the sequence of operations which realizes state transition $S_1 \rightarrow S_2 \rightarrow S_6 \rightarrow S_5 \rightarrow S_{11}$ is $RO_{II}(C_4, C_5) \rightarrow RO_{IV}(C_1) \rightarrow RO_{IV}(C_1) \rightarrow RO_{I}(C_1)$. When all type operations are used, we can derive 21 crossing states and 69 state transitions.

Thus, we can represent a manipulation process of a linear object as finite crossing states and their transitions. Moreover, we can plan it qualitatively when the initial state, the objective state, and several intermediate states in some cases are given.

## 4. Determination of Grasping Points and Their Moving Direction

In this section, we explain a procedure to determine grasping points and their moving direction in order to realize derived sequences of state transitions. We assume that manipulators grasp not a crossing point but a segment between two crossing points. Let us describe a segment between $C_i$ and $C_j$ as $L_{ip,jp}$ where $p$ indicates that the segment exists whether in front of ($p = f$) or behind ($p = b$) another segment at point $C_i$ and $C_j$. For example, a crossing region as shown in Fig. 4(a-2) consists of three segments: $L_{xp,ib}$, $L_{ib,if}$, and $L_{if,yp}$ where $x$ and $y$ mean the previous and the next crossing point number, respectively. We can move a segment by grasping at least one point in the segment. Then, grasping points can be described as combination of segments included in a crossing region as shown in Fig.4.

Next, let us consider the moving direction of a grasping point in order to realize basic operations. In Fig.4(b-2), by increasing the distance between both sides of the lower part, the crossing state can be changed into raveled one shown in Fig.4(b-1). Moving the middle of the lower part down can also change it. However, it is not predictable whether the current state can change or not when the right side of the lower part is twisted. Moreover, it is never changed by decreasing the distance between both sides of the lower part. Thus, realizability of a basic operation depends on the kind/direction of motion of grasping points.

We define four kinds of unit motion: translation along an axis which is parallel/perpendicular to the central axis of the object and rotation around an axis which is parallel/perpendicular to the central axis. Let us describe these unit motions as $T_c$, $T_p$, $R_c$, and $R_p$, respectively. Furthermore, we define the direction of unit motions as shown in Table 1. Then, by selecting feasible combinations of unit motions and grasping points, basic operations can be realized.

Moreover, we introduce quantified realizability of each crossing/raveling/arranging operation by each combination of unit motions and grasping points. For example, in Fig.4(a-2), motion $R_p^-$ of segment $L_{ib,if}$ can realize type-I raveling operation. So, its realizability are equal to 1. Motion $R_c^+$ of segment $L_{if,yp}$ may also realize the operation in some cases. So, we regard its realizability as be 0.5. However, it can not

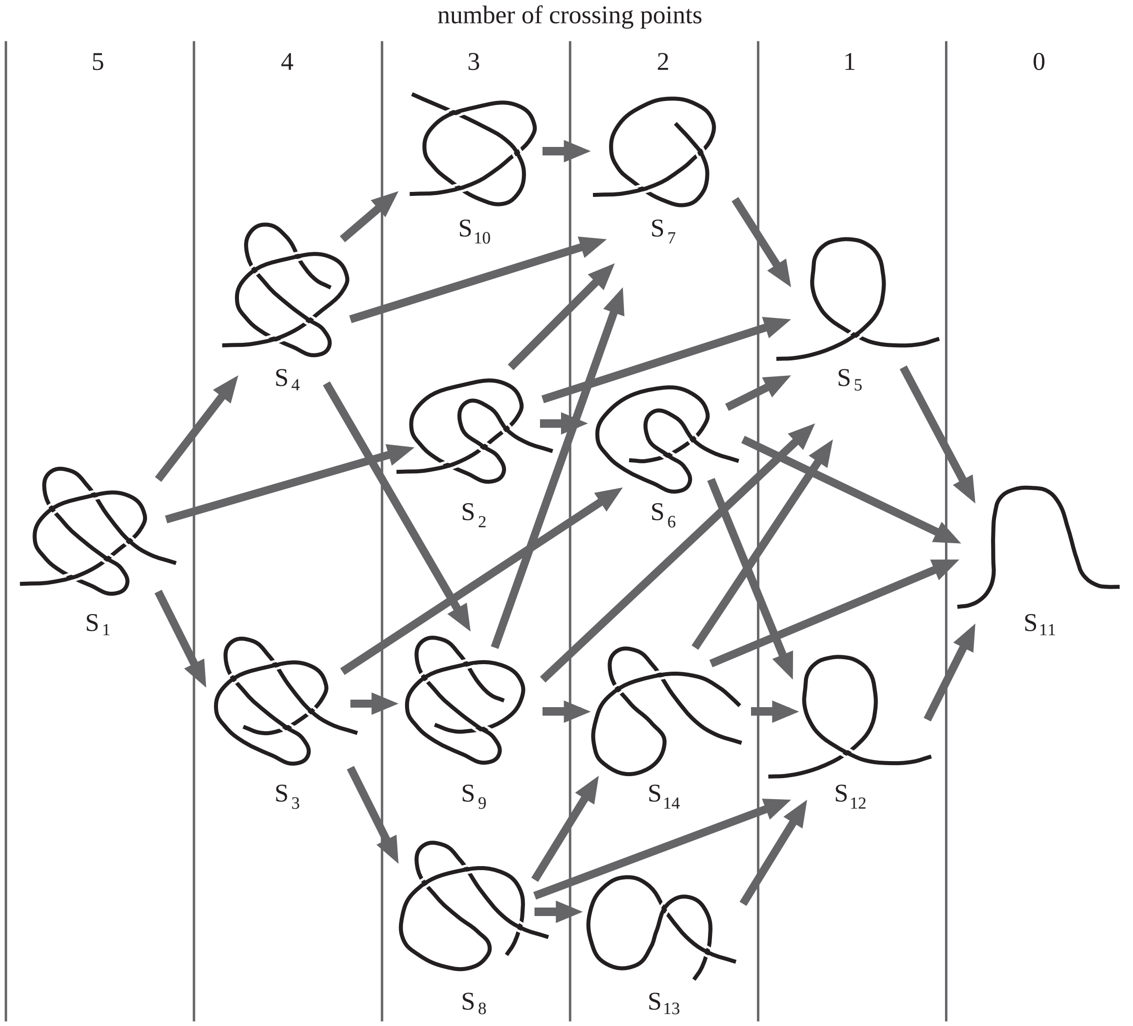



Fig. 6 Result of state transition network generation

be realized by motion $R_p^+$ of segment $L_{ib,if}$. So, its realizability becomes 0. By introducing such quantified realizability, we can exclude some combinations of unit motions and grasping points which can not realize operations at all. Furthermore, we can place the rest of combinations in descending order of realizability. Note that quantitative analysis is needed in order to determine actual realizability. Our introduced realizability in this section is one of criteria for evaluation and reduction of derived combinations of unit motions and grasping points in the qualitative analytical phase.

Thus, we can derive finite sequences of crossing state transitions of a linear object and feasible combinations of unit motions and grasping points with respect to each sequence, that is, rough manipulation plans.

## 5. Evaluation of Manipulation Processes

In this section, we introduce some criteria for evaluation of generated rough manipulation plans which is represented as sequences of crossing state transitions.

First, let us consider the number of grasping points in one crossing state; $N_m$. We assume that the number of available manipulators is limited. Then, $N_m$ must not exceed the number of manipulators in order to realize a transition sequence including such state.

Next, let us consider the number of state transitions through one sequence; $N_t$. In this study, we regard a sequence including fewer intermediate states, that is, fewer state transitions as be preferable.

Finally, let us consider changing times of grasping points through one sequence; $N_c$.

If a grasping point is changed during manipulation, the position/direction of a segment to be moved next must be estimated for the detailed planning. Furthermore, it takes more time for change of a grasping point. Therefore, a sequence in which grasping points are not changed frequently is preferable.

By using these criteria including realizability mentioned above, we can narrow down candidates for manipulation plans. After that, quantitative analysis should be executed in order to check whether such

Table 1 Direction of unit motions

| | $x = +$ | $x = -$ |
|---|---|---|
| $\mathrm{T}_c^x$ | translation in the same direction of the central axial vector | translation in the opposite direction of the central axial vector |
| $\mathrm{T}_p^x$ | translation closer to another part | translation away from another part |
| $\mathrm{R}_c^x$ | left hand helix torsion with respect to the central axial vector | right hand helix torsion with respect to the central axial vector |
| $\mathrm{R}_p^x$ | left hand helix torsion with respect to a perpendicular axial vector to the central axis | right hand helix torsion with respect to a perpendicular axial vector to the central axis |

manipulation can be realized practically or not when physical properties of a linear object such as rigidity are considered.

## 6. Case Study

In this section, we demonstrate the effectiveness of our developed manipulation process generation system using a proposed method in this paper.

Fig.7 shows a required manipulation. It corresponds to raveling of a overhand knot. The initial state shown in Fig.7(a) can be represented as $\mathrm{E}_l$-$\mathrm{C}_1^+$-$\mathrm{C}_2^+$-$\mathrm{C}_3^+$-$\mathrm{C}_1^+$-$\mathrm{C}_2^+$-$\mathrm{C}_3^+$-$\mathrm{E}_r$ and the objective state shown in Fig.7(b) can be represented as $\mathrm{E}_l$-$\mathrm{E}_r$. We assume that the left endpoint of a linear object is fixed through manipulation and the object are grasped its both endpoints in the initial state. A black rectangle and a black circle in Fig.7 represent the position of a fixure and a manipulator, respectively. Moreover, we assume that three manipulators including the previous one are available. Then, one sequence of state transitions: $\mathrm{RO_{IV}}(\mathrm{C}_3) \rightarrow \mathrm{RO_{IV}}(\mathrm{C}_2) \rightarrow \mathrm{RO_{I}}(\mathrm{C}_1)$ is derived by the system.

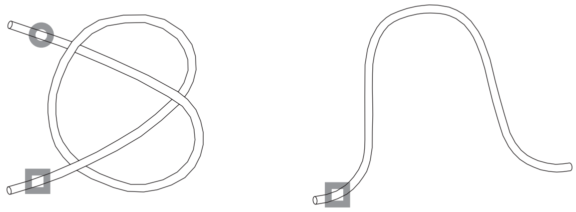
Furthermore, three manipulation plans illustrated in Fig.8 are selected by considering criteria $N_m$, $N_t$, and $N_c$. Combination of a unit motion and a grasping segment with respect to each plan are shown in Table 2. In plan 1, a grasping point is never changed. However, this plan involves the risk of entwining of a linear object with a manipulator. Let us regard a manipulator as a part of the linear object. Plan 1 can be then realized without entwining if the initial state of the object including the manipulator can be represented as shown in Fig.9(a) where gray regions in Fig.9 represents the manipulator. While, in the case of Fig.9(b), this resultant knot can not be raveled by plan 1. In plan 2 and plan 3, entwining can be avoided by taking note of approaching direction of the manipulator when it changes its grasping point.

Thus, our proposed method can be useful for the rough planning of linear object manipulation, especially knotting/raveling.

## 7. Toward Quantitative Planning

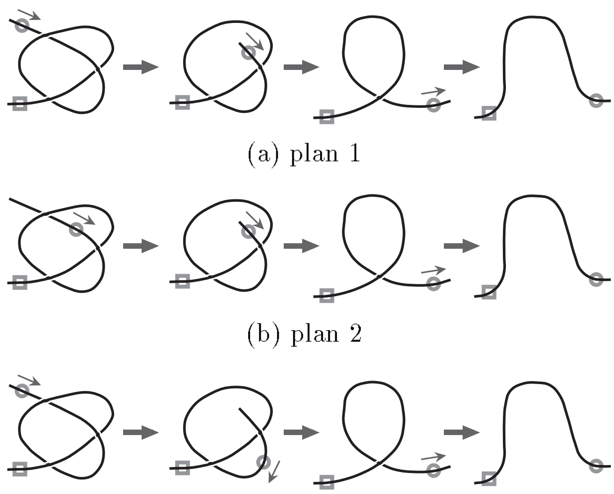
We can plan linear object manipulation qualitatively by applying our proposed method in the previous section. It is not enough to determine grasping points of manipulators or their trajectories in detail. However, we had also developed an analytical method to model the shape of a deformable linear object[5]. Fig.10 shows an numerical example. In this example, the central axis at both endpoints of a linear object is aligned in the initial state. Next, one endpoint is moved along this axis in order to shorten the distance between both endpoints.

The computed shape has one knot as the distance between the endpoints decreases. In this state, the object has not only bending deformation but also has torsional deformation because the potential energy in



(a) initial state (b) objective state

Fig. 7 Required manipulation – raveling of overhand knot



(a) plan 1

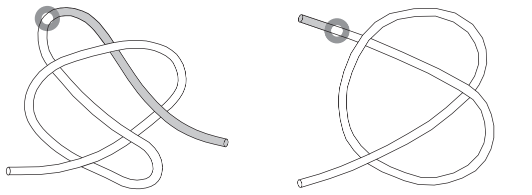
(b) plan 2

(c) plan 3

Fig. 8 Candidates for manipulation plans

Table 2 Grasped segment, unit motion, and evaluation values of each plan

| number of crossing points | 3→2 | 2→1 | 1→0 | $\max\{N_m\}$ | $N_t$ | $N_c$ |
|---|---|---|---|---|---|---|
| plan 1 | $T_c^-$ of $L_{3b,r}$ | $T_c^-$ of $L_{2a,r}$ | $T_c^+$ of $L_{1b,r}$ | 1 | 3 | 0 |
| plan 2 | $T_c^-$ of $L_{2a,3b}$ | $T_c^-$ of $L_{2a,r}$ | $T_c^+$ of $L_{1b,r}$ | 1 | 3 | 1 |
| plan 3 | $T_c^-$ of $L_{3b,r}$ | $T_c^-$ of $L_{1b,2a}$ | $T_c^+$ of $L_{1b,r}$ | 1 | 3 | 1 |



(a) possible to ravel (b) impossible to ravel

Fig. 9 Relationship between linear object and manipulator

this state is smaller than that when the object has only bending deformation. Thus, we can simulate linear object deformation including bend and twist.
By using a method proposed in the previous section, we can find feasible sequences of combinations of grasping points and their moving direction in order to realize a desired manipulation process. For more detail planning, the position of grasping points on the object and the direction of force/moment which should be imposed are required. If they can be determined, the geometrical shape of a linear object can be computed when various forces/moments are imposed on it. Therefore, it seems that the manipulation strategy can be derived automatically by combining a qualitative representation proposed in this paper with such a quantitative analysis.

Fig. 10 Computational result of linear object deformation

## 8. Conclusions

In this paper, a qualitative representation method of linear object manipulation was proposed toward its general manipulation planning.
Firstly, a representation method of the state of a linear object in three-dimensional space was proposed considering self-contact. It can be represented as finite crossing states which include the number of crossing points and the crossing type of each crossing point. Secondly, state transitions among those states were defined by introducing four kinds of basic operations. A state transition corresponds to a basic operation which changes the number of crossing points or permutates their sequence. Then, possible sequences of crossing state transitions, that is, possible manipulation processes can be generated when the initial state and the objective state are given. Thirdly, a method for determination of grasping points and their moving direction in order to realize derived manipulation processes was proposed. Furthermore, some criteria for evaluation of manipulation processes were introduced. Finally, that our proposed method can be applied to the rough planning of linear object manipulation was demonstrated.
It is expected that this method will be useful for the establishment of systematic approach to the planning of linear object manipulation.

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