# Planning of Knotting Manipulation

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**Abstract**
A planning method for knotting/unknotting manipulation of deformable linear objects is proposed. Firstly, topological states of a linear object are represented as finite sequence of crossings and two attributes of each crossing. Secondly, transitions among the topological states are defined. They correspond to operations that change the number of crossings or permutate their sequence. Then, we can generate possible sequences of crossing state transitions, that is, possible manipulation processes from an initial state to a given objective state. Thirdly, a method for determination of grasping points and their moving direction is proposed to realize derived manipulation processes. By using it, qualitative manipulation plans can be generated. Furthermore, criteria for evaluation of manipulation plans are introduced to reduce the candidates of manipulation plans. Finally, it is demonstrated that our developed system based on the above method can generate manipulation plans for tying an overhand knot.
**Keywords:** Linear Objects, Manipulation, Knotting, Unknotting, Planning

## 1. Introduction

Deformable linear objects such as tubes, cords, wires, and threads are used widely; not only for data transmission or for object transportation but also for fixing or packing of objects including themselves. Such manipulative tasks include knotting. On the other hand, self-entwining of linear objects should be avoided during their manipulative processes. Therefore, it is important for linear object manipulation to analyze knotting or entwining.

Hopcroft et al. have devised a grammar of knots to express various knotting manipulation[1]. Matsuno et al. realized a task of tying a cylinder with a rope by a dual manipulator system identifying the rigidity of the rope from visual information[2]. Morita et al. have been developing a system for knot planning from observation of human demonstrations[3]. In these studies, knotting manipulation of a linear object could be realized by a mechanical system, but how to knot is given.

To make a bowknot, for example, we manipulate a linear object dexterously by using several fingers of both hands for bending, twisting, holding, and/or binding. However, how to make a bowknot of us is not unique because it depends on our physical makeup and experience. If knotting/unknotting process of a linear object can be modeled, it is useful for design of knotting/unknotting system with mechanism unlike human arms/hands and for planning suitable for such system. Therefore, in this paper, we propose a planning method for knotting/unknotting of deformable linear objects.

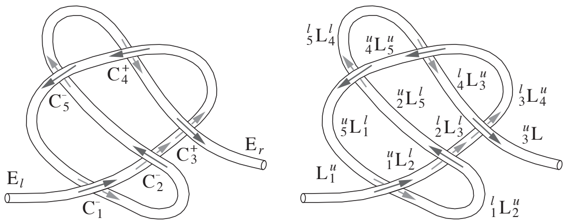
Firstly, topological states of a linear object are represented as finite sequence of crossings and two attributes of each crossing. Secondly, transitions among the topological states are defined. They correspond to operations that change the number of crossings or permutate their sequence. Then, we can generate possible sequences of crossing state transitions, that is, possible manipulation processes from an initial state to a given objective state. Thirdly, a method for determination of grasping points and their moving direction is proposed to realize derived manipulation processes. By using it, qualitative manipulation plans can be generated. Furthermore, criteria for evaluation of manipulation plans are introduced to reduce the candidates of manipulation plans. Finally, it is demonstrated that our developed system based on the above method can generate manipulation plans for tying an overhand knot.

## 2. Representation of Knotting/Unknotting Process

### 2.1 Crossing States

This section describes the crossing states of a deformable linear object. First, let us project the three-dimensional shape of a linear object on a plane. The projected two-dimensional curve may cross with itself. Crossings in the projected curve can specify the crossing state.

Next, let us number crossings along the projected curve from one endpoint to the other. One endpoint of the projected curve is defined as the left endpoint, and the other is defined as the right endpoint in this paper. Then, we can define the direction from the left endpoint to the right endpoint along the object as the counting direction. Fig.1 shows an example of a knotted linear object. This knot cor-




(a) crossings (b) segments

Fig. 1 Example of knotted linear object

(a) left-handed helical (b) right-handed helical

Fig. 2 Crossing type

responds to a slip knot. The object has 5 crossings and their sequence is denoted as $E_l$-$C_1$-$C_2$-$C_3$-$C_4$-$C_5$-$C_1$-$C_2$-$C_5$-$C_4$-$C_3$-$E_r$, where $E_l$, $E_r$, and $C_1$ through $C_5$ represent the left endpoint, the right endpoint, and crossings, respectively. In addition, whether each crossing is involved in the upper part or in the lower part is specified. Symbol $C_i^u$ describes the $i$-th crossing point is involved in the upper part while $C_i^l$ denotes it is in the lower part. Furthermore, we categorize the crossings into two: left-handed helical crossing illustrated in Fig.2(a) and right-handed helical crossing described in Fig.2(b). The upper part overlaps on the right side of the lower part at first and then on its left side in a left-handed helical crossing. The upper part overlaps on the left side of the lower part at first and then on its right side in a right-handed helical crossing. Symbol $C_i^-$ denotes the $i$-th crossing is left-handed helical while $C_i^+$ represents it is right-handed helical.

The list of symbols at individual crossing points determines the crossing states of a linear object. The crossing state of a knotted object shown in Fig.1(a) is described as $E_l$-$C_1^{u-}$-$C_2^{l-}$-$C_3^{l+}$-$C_4^{u+}$-$C_5^{u-}$-$C_1^{l-}$-$C_2^{u-}$-$C_5^{l-}$-$C_4^{l+}$-$C_3^{u+}$-$E_r$.

Let us describe a segment between $C_i$ and $C_j$ as ${}_i^p L_j^p$ where $p$ indicates whether the segment is an upper part (then $p$=$u$) or a lower part (then $p$=$l$) at crossing $C_i$ and $C_j$. A knotted object shown in Fig.1(b) has 11 segments. For example, the second segment between $C_1^{u-}$ and $C_2^{l-}$ is denoted as ${}_1^u L_2^l$. Terminal segments adjoining the left and the right endpoints are described as $L_i^p$ and ${}_i^p L$, respectively.

Consequently, we can represent the crossing states of a knotted linear object by a list of crossing point symbols. This representation is topological; no geometric properties such as its length and thickness nor physical properties such as its weight and rigidity are included.

### 2.2 Basic Operations for Crossing State Transitions

Knotting/unknotting process of a linear object corresponds to changing the number of its crossings. In this section, we introduce basic operations that perform the transitions among crossing states of a knotted object. In order to change the crossing state of a linear object, an operation must be performed on the object. Therefore, a state transition corresponds to an operation that changes the number of crossings or permutates their sequence. In this paper, four basic operations are prepared as shown in Fig.3. Operation I, II, and III are equivalent to Reidemeister move I, II, and III in the knot theory[4], respectively. Operations I, II, and III are applied to the intermediate of a linear object while operation IV manipulates an endpoint of the object. It is proved that any knot can be changed into another knot topologically equivalent to the original one by three Reidemeister moves alone in the knot theory. Operation IV is needed because a linear object has endpoints in general while the knot theory does not focus on the endpoints of the object. Operation I, II, and IV increase or decrease the number of crossings. Let us divide operation I into two: crossing operation $CO_I$ and uncrossing operation $UO_I$. Crossing operation $CO_I$ increases the number of crossings while uncrossing operation $UO_I$ decreases the number. Crossing operation $CO_{II}$ and $CO_{IV}$ and uncrossing operation $UO_{II}$ and $UO_{IV}$ are defined as well. Operation III does not change the number of crossings but change their sequence. Operation III is referred to as an arranging operation $AO_{III}$. Then, a manipulation process can be represented as transitions of crossing states. It corresponds to iteration of crossing, uncrossing, or arranging operations.

The number of possible crossing operations with respect to one crossing state can be larger than the number of possible uncrossing operations. It means that generation of possible manipulation processes from a crossed state to an uncrossed state is more effective than that from an uncrossed state to a crossed state because the more possible states/operations must be considered in the latter. In this paper, a state transition network is generated from a crossed state to an uncrossed state when the initial state and the objective state of a linear object are given. Thus, we can represent knotting/unknotting processes of a linear object as a network of finite crossing states and their transitions.

## 3. Motion Planning in Knotting/Unknotting Manipulation

In the previous section, knotting/unknotting process of a linear object is represented as a sequence of crossing state transitions. Moreover, we find that possible knotting/unknotting processes can be generated once the initial and the objective states are given. In order to accomplish one of possible processes, we have to grasp, move, and release the ob-

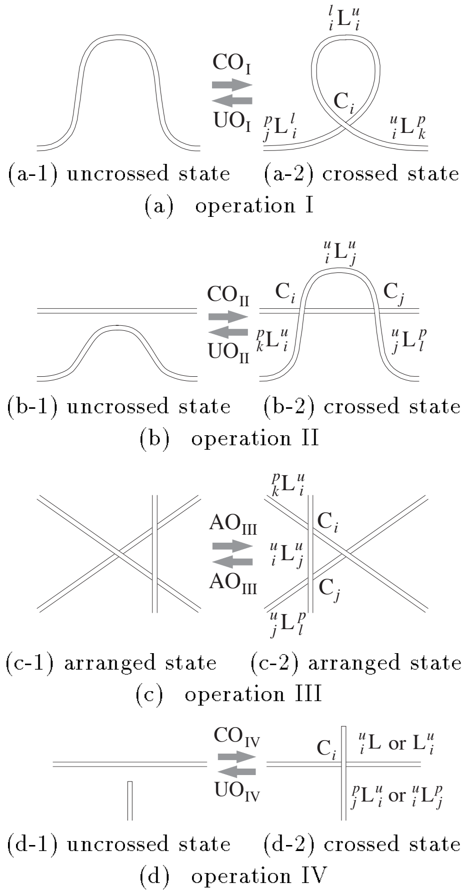



Fig. 3 Basic operations

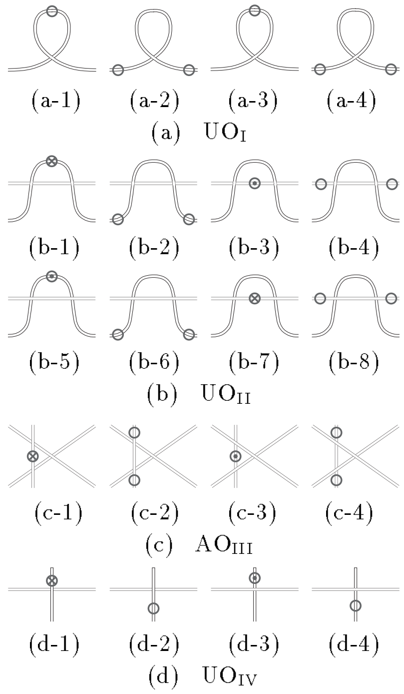



Fig. 4 Grasping points for uncrossing operations

ject during the processes. Whether the crossing state of the object changes as expected or not depends on grasping points and their moving direction. Let us define a qualitative manipulation plan as a sequence of crossing state transitions including grasping points and their moving direction to realize each state transition. In this section, we explain a procedure to determine adequate grasping points and their moving direction for one state transition. In addition, for detailed planning and actual execution of manipulation, we have to narrow down qualitative manipulation plans. Therefore, we introduce criteria to evaluate qualitative manipulation plans.

### 3.1 Actions for Uncrossing Operations

Uncrossing operations delete a crossing by moving its upper part or lower part. Assume that a manipulator grasps a segment between two neighboring crossing points during uncrossing operations. A crossed state shown in Fig.3(a-2) consists of three segments: $^p_j L^l_i$, $^l_i L^u_i$, and $^u_i L^p_k$. Segment $^l_i L^u_i$ is deleted by uncrossing $C_i$. In this case, segment $^l_i L^u_i$ is referred to as a target segment of the uncrossing. We assume that a target segment or its adjacent segments in each crossed state should be grasped in order to realize each uncrossing operation. For example, in Fig.3(a-2), segment $^l_i L^u_i$ or segments $^p_x L^l_i$ and $^u_i L^p_y$ should be grasped.

Furthermore, we define the approaching direction of a manipulator with respect to the projection plane: from the front side or the back side. Whether each operation can be performed or not depends on this direction. For example, in Fig.3(d-2), a manipulator can not perform $UO_{IV}$ when it grasps the terminal segment $^u_i L$ from the back side. Some operations can be performed if a manipulator approaches to an appropriate segment from the front side and grasps the segment. Others can be performed if a segment is grasped from the back side. The others do not care with the approaching direction of a manipulator. A set of grasping points and their approaching direction for each uncrossing operation is illustrated in Fig.4. A circle with dot, a circle with cross, and a open circle represent a point to be grasped from the front side, the back side, and whichever side, respectively.

Next, let us consider moving direction of a grasping point to realize each operation. Assume that a pair of fingertips grasps a linear object during its knotting/unknotting. Once the fingertip pair grasps the object firmly, it can be regarded as a rigid body. Generally, a rigid body in the three-dimensional space has three DOF in translation and three DOF in rotation. Note that the translation along the projection normal does not change the crossing state of a linear object. Omitting this translation, we apply two DOF in translation along the projection plane and three DOF in rotation into the knotting/unknotting of a linear object. Then, we can select a set of grasping points and their corresponding DOF to perform individual basic operations. In this paper, this set is

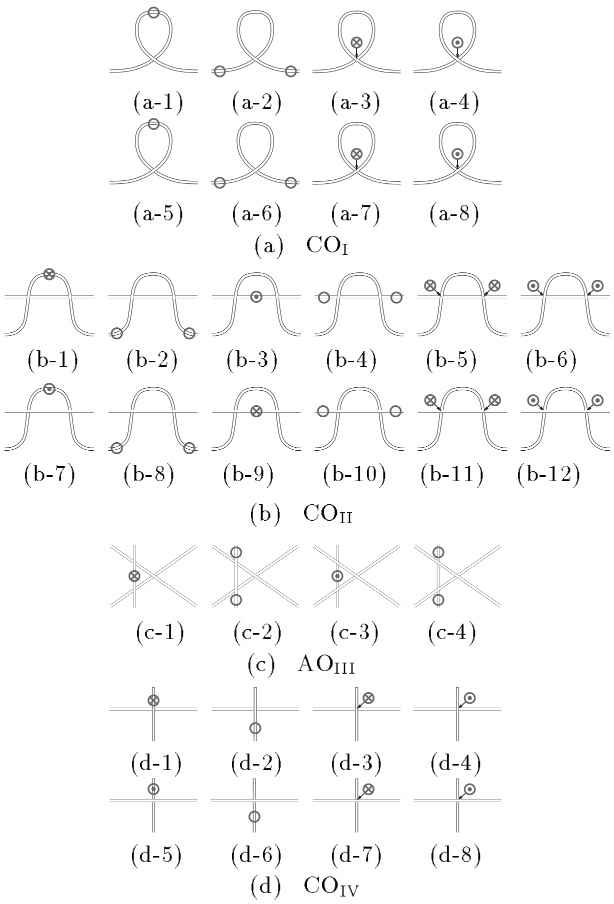



Fig. 5 Grasping points for crossing operations

referred to as an *action*.

### 3.2 Actions for Crossing Operations

In crossing operations, not only a segment but also a point to be crossed can be grasped. Therefore, 32 sets of grasping points shown in Fig.5 can be derived. Fig.5 shows crossed states after crossing operations.

Thus, actions, that is, adequate sets of grasping points and their corresponding DOF to realize each crossing/uncrossing operation can be determined. Consequently, possible qualitative manipulation plans, that is, sequences of crossing state transitions and actions for each state transition, can be generated by a computer system when the initial and the objective crossing state of a linear object are given.

All crossing state of a knotted linear object has at least one crossing nearest to one endpoint. This crossing can be deleted by operation $UO_{IV}$. This implies that any knot can be unknotted by operation $UO_{IV}$ alone. We can also make any knot by repeating operation $CO_{IV}$. Moreover, if we select a grasping point shown in Fig.4(d-2), Fig.4(d-4), Fig.5(d-3), or Fig.5(d-7), operation $CO_{IV}/UO_{IV}$ can be performed by one manipulator approaching from the front side of the projection plane. It means that any knotting/unknotting can be realized by one-handed manipulation[5].

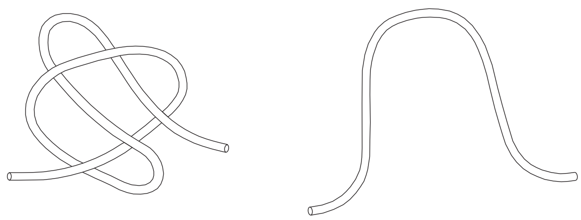



Fig. 6 Example of required manipulation

### 3.3 Evaluation of Manipulation Plans

we introduce criteria to evaluate generated qualitative manipulation plans.

First, let $N_t$ be the number of state transitions through one sequence. In this paper, we prefer a sequence including fewer intermediate states, that is, fewer state transitions because it takes much time to finish the required manipulation when a selected sequence includes many state transitions. Note that a knotting/unknotting process corresponds to increase/decrease of crossings of a linear object. Recall that operation II generates/deletes two crossings while operation I and IV generates/deletes one crossing. Then, we find that a sequence including the more operations II consists of the fewer intermediate states.

Next, let $N_c$ be the changing times of grasping points through one sequence. When a grasping point never change during manipulation, position and direction of a linear object at the grasping point corresponds to those of fingertips of a manipulator obviously. So, estimation of the object shape is not needed once the manipulator grasps the object. However, if a grasping point changes during manipulation, position and direction of a segment to be grasped in the next operation must be estimated in the detailed planning. Furthermore, it takes much time to change a grasping point. Therefore, a sequence in which grasping points are not changed frequently is preferable.

## 4. Example of Possible Process Generation

In this section, we shows an example of possible unknotting process generation by a computer system. Fig.6 shows a required manipulation. It corresponds to untying a slip knot. The initial state in Fig.6(a) is represented as $E_l\text{-}C_1^{u-}\text{-}C_2^{l-}\text{-}C_3^{l+}\text{-}C_4^{u+}\text{-}C_5^{u-}\text{-}C_1^{l-}\text{-}C_2^{u-}\text{-}C_5^{l-}\text{-}C_4^{l+}\text{-}C_3^{u+}\text{-}E_r$ and the objective state in Fig.6(b) is represented as $E_l\text{-}E_r$. Assuming that only uncrossing operations can be used, that is, without $AO_{III}$, 14 crossing states and 32 state transitions are derived as shown in Fig.7. Including operation $AO_{III}$, we can derive 21 crossing states and 69 state transitions. Thus, possible knotting/unknotting processes of a linear object can be generated automatically when the initial and the objective states are given.

The number of crossings in the initial state is five, and that in the objective state is zero. We

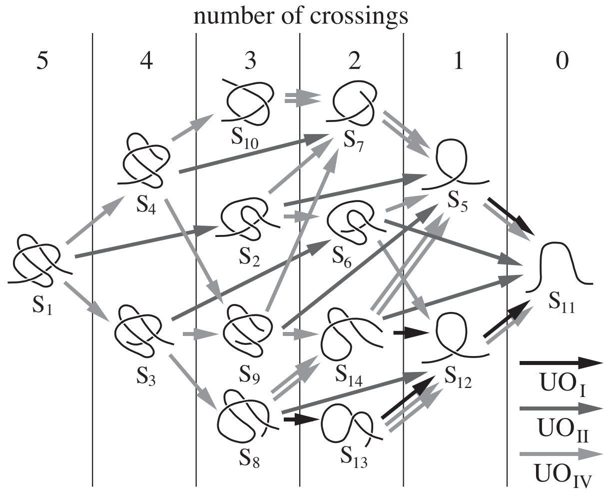



Fig. 7 Result of manipulation process planning

can reduce the number of crossings from five to zero by applying two operations $\mathrm{UO_{II}}$ and one operation $\mathrm{UO_I/UO_{IV}}$ at least. Their possible sequences are described as follows :

$$\mathrm{OQ_1} \ : \ \mathrm{UO_{II}} \rightarrow \mathrm{UO_{II}} \rightarrow \mathrm{UO_I/UO_{IV}}, \qquad (1)$$

$$\mathrm{OQ_2} \ : \ \mathrm{UO_{II}} \rightarrow \mathrm{UO_I/UO_{IV}} \rightarrow \mathrm{UO_{II}}, \qquad (2)$$

$$\mathrm{OQ_3} \ : \ \mathrm{UO_I/UO_{IV}} \rightarrow \mathrm{UO_{II}} \rightarrow \mathrm{UO_{II}}. \qquad (3)$$

Then, we check whether the required process can be realized by applying these three uncrossing operations to the object in the above orders or not. In Fig.7, the following sequences of state transitions $\mathrm{SQ_1}$, $\mathrm{SQ_2}$, and $\mathrm{SQ_3}$ correspond to operation sequences $\mathrm{OQ_1}$, $\mathrm{OQ_2}$, and $\mathrm{OQ_3}$, respectively.

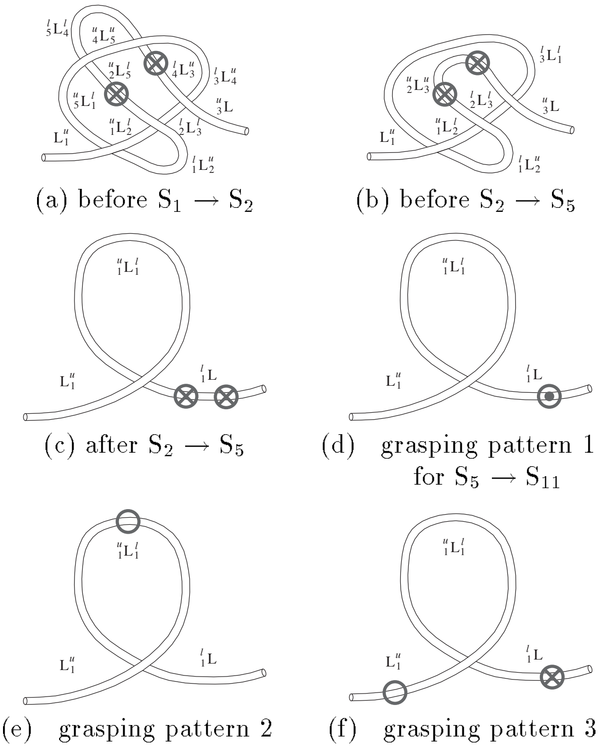
$$\mathrm{SQ_1} \ : \ \mathrm{S_1} \rightarrow \mathrm{S_2} \rightarrow \mathrm{S_5} \rightarrow \mathrm{S_{11}}, \qquad (4)$$

$$\mathrm{SQ_2} \ : \ \mathrm{S_1} \rightarrow \mathrm{S_2} \rightarrow \mathrm{S_6} \rightarrow \mathrm{S_{11}}, \qquad (5)$$

$$\mathrm{SQ_3} \ : \ \mathrm{S_1} \rightarrow \mathrm{S_3} \rightarrow \mathrm{S_6} \rightarrow \mathrm{S_{11}}. \qquad (6)$$

If the required process can not be realized with two operations $\mathrm{UO_{II}}$ and one operation $\mathrm{UO_I/UO_{IV}}$, we check that with one operation $\mathrm{UO_{II}}$ and three operations $\mathrm{UO_I/UO_{IV}}$. Thus, we can derive manipulation processes including fewer state transitions, that is, with smaller $N_t$ even if we do not generate the whole state transition network.

Next, we select adequate actions so that a manipulation process has fewer changing times $N_c$ of grasping points. Let us consider sequence $\mathrm{SQ_1}$. For the first transition from state $\mathrm{S_1}$ to state $\mathrm{S_2}$, assume that segments ${}^{u}_{2}\mathrm{L}^{l}_{5}$ and ${}^{l}_{4}\mathrm{L}^{u}_{3}$ are grasped from the front side as shown in Fig.8(a) and moved to perform operation $\mathrm{UO_{II}}$. Then, grasped segments become equivalent to segment ${}^{u}_{2}\mathrm{L}^{u}_{3}$ in state $\mathrm{S_2}$ as shown in Fig.8(b). State $\mathrm{S_2}$ can be changed into state $\mathrm{S_5}$ by moving segment ${}^{u}_{2}\mathrm{L}^{u}_{3}$. After that, it is found that segment ${}^{l}_{1}\mathrm{L}$ in state $\mathrm{S_5}$ is grasped from the front side by two manipulators as shown in Fig.8(c). There are three ways



(a) before $\mathrm{S_1} \rightarrow \mathrm{S_2}$ (b) before $\mathrm{S_2} \rightarrow \mathrm{S_5}$

(c) after $\mathrm{S_2} \rightarrow \mathrm{S_5}$ (d) grasping pattern 1 for $\mathrm{S_5} \rightarrow \mathrm{S_{11}}$

(e) grasping pattern 2 for $\mathrm{S_5} \rightarrow \mathrm{S_{11}}$ (f) grasping pattern 3 for $\mathrm{S_5} \rightarrow \mathrm{S_{11}}$

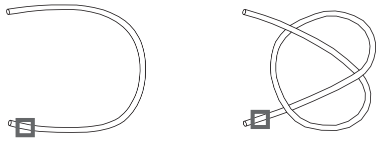
Fig. 8 Grasping patterns and their changing times

to change the state into $\mathrm{S_{11}}$. The first is to regrasp segment ${}^{l}_{1}\mathrm{L}$ from the back side for operation $\mathrm{UO_{IV}}$ as shown in Fig.8(d). The second is to release segment ${}^{l}_{1}\mathrm{L}$ and to grasp segment ${}^{u}_{1}\mathrm{L}^{l}_{1}$ as shown in Fig.8(e) for operation $\mathrm{UO_I}$ or for operation $\mathrm{UO_{IV}}$. The third is to grasp segment $\mathrm{L}^{u}_{1}$ keeping segment ${}^{l}_{1}\mathrm{L}$ grasped for operation $\mathrm{UO_I}$ as shown in Fig.8(f). Anyway, we have to change grasping points for the last transition from state $\mathrm{S_5}$ to state $\mathrm{S_{11}}$. Consequently, in the above plans to perform sequence $\mathrm{SQ_1}$, $N_c = 1$ and it is minimum. We can also derive minimum $N_c$ for sequence $\mathrm{SQ_2}$ and sequence $\mathrm{SQ_3}$. The former is $N_c = 2$ and the latter is $N_c = 1$. This implies that sequence $\mathrm{SQ_2}$ should be eliminated from adequate manipulation plans. Thus, we can narrow down candidates of manipulation plans by considering $N_t$ and $N_c$. After that, quantitative analysis should be performed in order to check whether a selected manipulation can be realized practically or not considering physical properties of a linear object such as rigidity.

## 5. Case Study

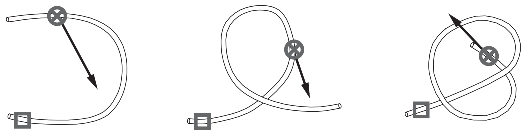
We demonstrate the effectiveness of our proposed method in this paper. Our developed system for knotting/unknotting manipulation consists of a PC, a 6 DOF manipulator, and a CCD camera. A linear object, whose physical properties are unknown, is laid on a table and its shape is captured by the camera fixed above the table. The table corresponds to the projection plane.

Fig.9 shows a required manipulation. It corre-



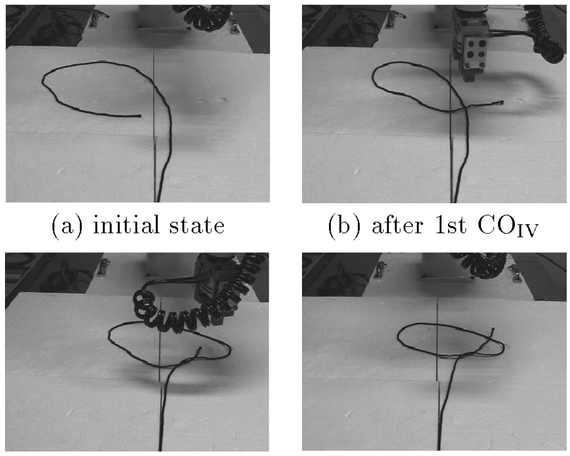
(a) initial state (b) objective state

Fig. 9 Required manipulation – tying overhand knot



(a) 1st $CO_{IV}$ (b) 2nd $CO_{IV}$ (c) 3rd $CO_{IV}$

Fig. 10 Generated manipulation plan



(a) initial state (b) after 1st $CO_{IV}$

(c) after 2nd $CO_{IV}$ (d) after 3rd $CO_{IV}$

Fig. 11 Result of manipulation

sponds to tying an overhand knot. The initial state shown in Fig.9(a) can be represented as $E_l$-$E_r$ and the objective state shown in Fig.9(b) can be represented as $E_l$-$C_1^{l+}$-$C_2^{u+}$-$C_3^{l+}$-$C_1^{u+}$-$C_2^{l+}$-$C_3^{u+}$-$E_r$. Assumptions of this case study are as follows:

- The left endpoint of the object is fixed during manipulation; an open square in Fig.9 indicates the position of a fixure.
- The manipulator releases the object whenever one crossing operation is finished. It implies that criterion $N_c$ is not considered.
- We use operation $CO_{IV}$ alone because any operation $CO_{IV}$ can be performed by one manipulator approaching from the front side of the projection plane, that is, from above the table.

Then, one sequence of crossing state transition shown in Fig.10 is generated. It consists of three operations $CO_{IV}$.

Next, the system recognizes the current crossing state of the object from a gray-scale image. The position of individual crossings can be identified by analyzing the image. In this experiment, information about which part is up at each crossing is given for simplicity. However, it can be obtained automatically using a stereo camera. As adequate moving distance for a state transition is unknown, the system checks whether its crossing state is changed or not after moving the object. Thus, the manipulator can grasp, move, and release the object according to the generated qualitative plan. Fig.11 shows the result of this manipulation. Thus, we conclude that our proposed method is useful for planning of knotting/unknotting manipulation of deformable linear objects.

## 6. Conclusions

A planning method for knotting/unknotting manipulation of deformable linear objects was proposed. Firstly, knotting/unknotting processes of a linear object was represented as a sequence of finite crossing state transitions. Secondly, grasping points and their moving direction to perform each state transition were defined. Then, possible qualitative manipulation plans can be generated by a computer system when the initial state and the objective state of a linear object are given. Thirdly, criteria for evaluation of generated manipulation plans were introduced. By considering them, we can narrow down candidates of manipulation plans. Finally, an experiment for tying an overhand knot by our developed system based on the above method was shown.

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