Static Analysis of Deformable Object Grasping Based on Bounded Force Closure

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Abstract

A static analysis of deformable object grasping based on bounded force closure is presented. There are many manipulative operations that deal with deformable objects in manufacturing processes. Manipulative operations for these objects are often performed by utilizing their deformation actively while the operations may result in failure because of unexpected deformation of the objects during the manipulation process. In order to perform the manipulative operations for deformable objects successfully, it is necessary to evaluate their deformation by building object models and to derive task strategies by analyzing manipulation processes using the object models.

In this paper, we will analyze stable grasping of deformable objects based on the concept of bounded force closure. Firstly, we will introduce the concept of bounded force closure, which is an extension of force closure condition. Secondly, we will investigate the necessary condition for bounded force closure in order to derive the properties of bounded force closure grasping. Thirdly, we will formulate the deformation of linear objects as an example of deformable objects and we will propose a procedure to evaluate stability of deformable object grasping. Finally, some numerical examples will be shown in order to demonstrate the effectiveness of our proposed method.

1 Introduction

Many manufacturing processes that deal with deformable objects such as rubber tubes, sheet metals, cords, leather products, and paper sheets have been automated but most handling and manipulative processes are still done by humans. Automatic handling and manipulation of deformable objects are thus eagerly required. Manipulative operations for deformable objects are often performed by utilizing their deformation actively while the operations may result in failure because of unexpected deformation of the objects during the manipulation process. Evaluation of the deformation of these objects on a computer in advance using the object models is thus necessary so that we can perform the manipulative operations successfully.

Furthermore, in order to perform such manipulative operation successfully, it is important to derive task strategies by analyzing the manipulative processes using deformable object models. Grasping is one of basic operation in the manufacturing processes. Two concepts for evaluation of stability in grasping of rigid objects, form closure[1] and force closure[2] have been analyzed. In grasp analysis, contacts between fingers and objects are analyzed extensively[3][4]. Analysis and planning of manipulation using the theory of polyhedral convex cones has reported[5]. However, these analyses assume that objects to grasp are rigid and do not mention the case that objects are deformable.

In this paper, we will analyze stable grasping of deformable objects based on the concept of bounded force closure. Firstly, we will introduce the concept of bounded force closure, which is an extension of force closure. Secondly, we will investigate the necessary condition for bounded force closure and will derive the properties of bounded force closure grasping. Thirdly, we will formulate the deformation of linear objects as an example of deformable objects and will propose a procedure to evaluate stability in deformable object grasping. Finally, some numerical examples will be shown in order to demonstrate the effectiveness of our proposed method.

2 Formulation of Deformable Object Grasping

Concepts of form closure and force closure have been proposed for evaluating stable grasping of rigid objects. In this section, we will examine whether these concepts can be applied to grasping of deformable objects or not and will introduce a new concept for deformable object grasping.

First, let us examine whether the form closure condition can be applied to the grasping of deformable objects. Form closure has been defined as to constrain all degrees of freedom of object motion. Note that the degree of freedom of a deformable object can be regarded to be infinite. Thus, all of the freedom cannot be constrained by a finite set of fingers. This implies that the concept of form closure cannot be applied to the grasping of deformable objects.

Next, we will examine whether the force closure condition can be applied to the grasping of deformable objects. Consider that a force and moment $\boldsymbol{p}$ is imposed on an arbitrary point of an object. If an arbitrary force and moment $\boldsymbol{p}$ is balanced with reaction forces from fingers around the object, location of the fingers with respect to the object satisfies the condition for force closure. Assuming that a rigid object contacts with t fingers, the force closure condition can be described as follows:

$$\begin{aligned} &\forall\, \boldsymbol{p}, \quad \exists\, R_1,\, R_2,\, \cdots,\, R_t \geq 0, \\ &\text{s.t.} \quad \boldsymbol{p} + \boldsymbol{F}_b + \sum_{i=1}^{t} R_i \boldsymbol{w}_i = \mathbf{o} \end{aligned} \tag{1}$$

where $\boldsymbol{F}_b$ is a body force, $\boldsymbol{w}_i$ is a wrench vector at the i-th contact point, and R_i is the magnitude of the i-th wrench. Let us investigate whether the above condition can be satisfied in the grasping of deformable objects. Note that the i-th finger exerts a constant grasping force at the i-th contact point in the stable grasping. Let us apply an external force at the i-th contact point in the same direction of the i-th grasping force. When the magnitude of the external force exceeds the magnitude of the grasping force, the contact between the deformable object and the i-th finger is lost. This implies that the above condition is never satisfied for the grasping of deformable objects. Furthermore, it is found that the magnitude of the forces is bounded for the stable grasping of deformable objects. Let us introduce the following concept for the grasping of deformable objects. Assume that the force $f\boldsymbol{n}$ whose magnitude is f and whose direction is $\boldsymbol{n}$ is imposed at a certain point $\boldsymbol{x}$ of the object. The wrench corresponding to this force is given by $\boldsymbol{w}_f = [\, \boldsymbol{n}^{\mathrm{T}},\ (\boldsymbol{x} \times \boldsymbol{n})^{\mathrm{T}}\,]^{\mathrm{T}}$. From the above discussion, we find that the force closure condition is satisfied unless the magnitude f exceeds a certain bound, which depends on position $\boldsymbol{x}$ and direction $\boldsymbol{n}$. Let $F(\boldsymbol{x},\, \boldsymbol{n})$ be the upper bound of magnitude f and F_{bfc} be the smallest value of $F(\boldsymbol{x},\, \boldsymbol{n})$:

$$F_{\mathrm{bfc}} = \min_{\boldsymbol{x}} \min_{\boldsymbol{n}} F(\boldsymbol{x}, \boldsymbol{n}).$$

Note that the force closure condition is satisfied for arbitrary $\boldsymbol{x}$ and arbitrary $\boldsymbol{n}$ unless the magnitude of an external force exceeds F_{bfc}. Let us call this condition *bounded force closure*. From the above discussion, the condition for bounded force closure can be described as follows:

$$\begin{aligned} &\forall\, \boldsymbol{x}, \quad \forall\, \boldsymbol{n}, \quad \exists\, F_{\mathrm{bfc}} > 0,\ R_1,\, R_2,\, \cdots,\, R_t \geq 0, \\ &\text{s.t.} \quad f\boldsymbol{w}_f + \boldsymbol{F}_b + \sum_{i=1}^{t} R_i \boldsymbol{w}_i = \mathbf{o}, \quad \forall\, f \in [\, 0,\ F_{\mathrm{bfc}}\,] \end{aligned} \tag{2}$$

where we call F_{bfc} allowable maximum external force.

Assume that the moment whose magnitude is m and whose rotational axis is given by $\boldsymbol{d}$ is imposed on the object. The wrench corresponding to this moment is given by $\boldsymbol{w}_m = [\mathbf{o}^{\mathrm{T}}, \boldsymbol{d}^{\mathrm{T}}]^{\mathrm{T}}$. Note that if magnitude m is smaller than a certain value, the condition for force closure is satisfied. Let $M(\boldsymbol{d})$ be the upper bound of magnitude m and M_{bfc} be the smallest value of $M(\boldsymbol{d})$:

$$M_{\mathrm{bfc}} = \min_{\boldsymbol{d}} M(\boldsymbol{d}).$$

We find that the force closure condition is satisfied for arbitrary $\boldsymbol{d}$ unless the magnitude of an external moment exceeds M_{bfc}. Therefore, an additional condition for bounded force closure can be described as follows:

$$\begin{aligned} &\forall\, \boldsymbol{d}, \quad \exists\, M_{\mathrm{bfc}} > 0,\ R_1,\, R_2,\, \cdots,\, R_t \geq 0, \\ &\text{s.t.} \quad m\boldsymbol{w}_m + \boldsymbol{F}_b + \sum_{i=1}^{t} R_i \boldsymbol{w}_i = \mathbf{o}, \quad \forall\, m \in [\, 0,\ M_{\mathrm{bfc}}\,] \end{aligned} \tag{3}$$

where we call M_{bfc} allowable maximum external moment.

When both conditions given by eqs.(2) and (3) are satisfied, we can prove that grasping of a deformable object is stable against arbitrary external force/moment, $[\, \boldsymbol{f}^{\mathrm{T}},\ \boldsymbol{m}^{\mathrm{T}}\,]^{\mathrm{T}}$, if the magnitude of $\boldsymbol{f}$ is smaller than F_{bfc} and the magnitude of $\boldsymbol{m}$ is smaller than M_{bfc}.

3 Evaluation of Stability in Deformable Object Grasping

In this section, we will analyze the bounded force closure condition given by eqs.(2) and (3). Let $\boldsymbol{w}_1$ through $\boldsymbol{w}_m$ be wrench vectors imposed on a deformable object by fingers. The region of resultant reaction forces is then given by

$$W = \left\{ \boldsymbol{w} = \sum_{i=1}^{t} R_i \boldsymbol{w}_i \;\middle|\; R_1, \cdots, R_t \geq 0 \right\}.$$

Eq.(2) is then rewritten as follows:

$$-\, f\boldsymbol{w}_f - \boldsymbol{F}_b \in W, \quad \forall f \in [\, 0,\ F_{\mathrm{bfc}}\,]. \tag{4}$$

The relationship between body force $\boldsymbol{F}_b$ and region W can be classified into three cases:

$$\begin{aligned} &\textbf{case 1} &:& \quad -\boldsymbol{F}_b \notin W \\ &\textbf{case 2} &:& \quad -\boldsymbol{F}_b \in \partial W \\ &\textbf{case 3} &:& \quad -\boldsymbol{F}_b \in W^{\mathrm{int}} \end{aligned}$$

where ∂W denotes the boundary of region W and W^{int} denotes the interior of region W.

Let us explain the above relationship using a simple 2D example illustrated in Figure 1. In this example, region W consists of two wrench vectors, $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$. Wrenches $-\boldsymbol{F}_{b1}$, $-\boldsymbol{F}_{b2}$, and $-\boldsymbol{F}_{b3}$ corresponding to case 1, 2, and 3, respectively. When $\boldsymbol{F}_b$ coincides $\boldsymbol{F}_{b1}$ or $\boldsymbol{F}_{b2}$, $-f\boldsymbol{w}_f - \boldsymbol{F}_b$ is not involved in region W for an infinitesimal positive value f along a certain wrench $\boldsymbol{w}_f$. This implies that eq.(4) is not satisfied in case 1 or case 2. Therefore, bounded force closure grasping must satisfy the following equation:

$$-\, \boldsymbol{F}_b \in W^{\mathrm{int}}. \tag{5}$$

We can prove that the above condition must be satisfied in general.

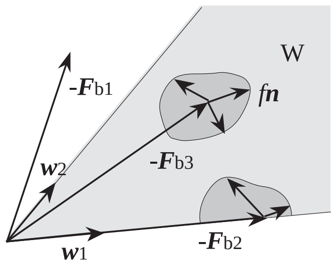


Figure 1: Condition for bounded force closure

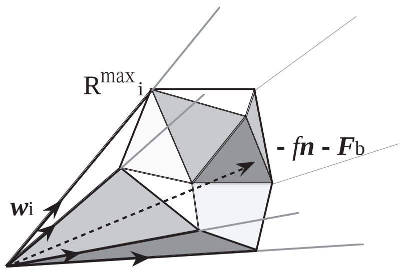


Figure 2: Region for bounded force closure

The interior set $W^{\rm int}$ is empty when the dimension of region W is less than six. This implies that a necessary condition for eq.(4) is given by

$$\dim W = 6.$$

Let us consider a case where the body force $\boldsymbol{F}_b$ is equal to zero. In this case, it can be proved that region W must coincide to the whole region R^6. Namely,

$$W = \mathrm{R}^6 \quad \text{if} \quad \boldsymbol{F}_b = \mathbf{o}.$$

From eq.(2), it turns out that R_i is involved in $[0,\ F_{\rm bfc}]$ if $\boldsymbol{F}_b = \mathbf{o}$ and $-\boldsymbol{n} = \boldsymbol{w}_i$ are satisfied because $f\boldsymbol{w}_f + R_i\boldsymbol{w}_i = \mathbf{o}$. Therefore, a non-negative coefficient R_i with respect to a certain wrench vector $\boldsymbol{w}_i$ upper bounded:

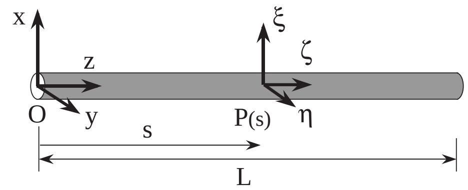
$$0 \leq R_i \leq R^{\rm max}{}_i, \quad i = 1, \cdots, t.$$

From the above discussion, we find that the set of wrenches that satisfy the bounded force closure condition given by eqs.(2) and (3) can be illustrated as Figure 2. Recall that the region of wrenches that satisfy the force closure condition given by eq.(1) can described by a semi-finite region, which is referred to as a polyhedral convex cone[5]. The region of wrenches that satisfy the bounded force closure condition can be described by a finite region shown in the figure rather than a semi-finite region.

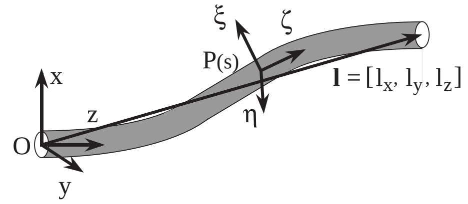
4 Modeling of Linear Object Deformation

4.1 Formulation of Deformation

In this section, we will briefly explain the formulation of the deformation of a linear object in three-dimensional space. In detail, see [6]. In addition,



(a) natural shape



(b) deformed shape

Figure 3: Coordinates systems describing object deformation

we will propose a procedure to compute the allowable maximum external force $F_{\rm bfc}$ and allowable maximum external moment $M_{\rm bfc}$.

First, we will represent the deformation of a linear object and its potential energy. Let L be the length of the object along its central axis and s be the distance from one endpoint of the object along its central axis. We will introduce the global space coordinate system and the local object coordinate systems at individual points on the object, as shown in Figure 3. Let $O-xyz$ be the coordinate system fixed on space and $P-\xi\eta\zeta$ be the coordinate system fixed on an arbitrary point $P(s)$ of the object. Select the direction of the local coordinate system $P-\xi\eta\zeta$ so that ζ-axis is aligned with the central axis of the object. Let us describe the orientation of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(s)$, $\theta(s)$, and $\psi(s)$, and let us introduce extensional strains at individual points $P(s)$ on the object along its central axis. Then, the spatial coordinates $\boldsymbol{x}(s)$ corresponding to point $P(s)$ and the potential energy U can be represented by use of these four variables, $\phi(s)$, $\theta(s)$, $\psi(s)$, and $\varepsilon(s)$. We will adopt a basic principle that the difference between the potential energy of a deformable object and the work done by external force/moment reaches to its minimum under the constraints imposed on the object at its stable deformed shape.

Let $\boldsymbol{p}_k$ be an external force/moment and $\delta\boldsymbol{q}_k$ be a displacement of point $P(s_k)$ when an external force/moment is imposed on the object at that point. Then, work W_k done by the external force/moment $\boldsymbol{p}_k$ can be described as follows:

$$W_k = \boldsymbol{p}_k \cdot \delta\boldsymbol{q}_k \tag{6}$$

The displacement $\delta\boldsymbol{q}_k$ can be also represented by use of the four variable, $\phi(s)$, $\theta(s)$, $\psi(s)$, and $\varepsilon(s)$.

Next, we will derive the geometric constraints imposed on the object. Constraints with respect to the position and the orientation of the object can be described by the following equations and inequalities:

$$g_i(\boldsymbol{x}) \leq 0, \quad i = 1, \cdots, u \tag{7}$$
$$h_j(\boldsymbol{x}) = 0, \quad j = 1, \cdots, v \tag{8}$$

These equations and inequalities are functions of $\phi(s)$, $\theta(s)$, $\psi(s)$, and $\varepsilon(s)$ as well because the spatial coordinate $\boldsymbol{x}$ is a function of these four variables.

From the above discussion, the shape of a deformed linear object is derived by minimizing the energy $U - \sum W_k$ under the equational and the inequality conditions given by eqs.(7) and (8).

4.2 Procedure to Compute Deformed Shape

As mentioned in the previous section, the shape of a deformed linear object is derived by minimizing the difference between potential energy and work done by external force and moment under geometric constraints. Thus, computation of the deformed shape of a linear object results in a variational problem. In this section, we will propose a procedure to compute the allowable maximum external force $F_{\rm bfc}$ and the allowable maximum external moment $M_{\rm bfc}$. Let us express functions $\phi(s)$, $\theta(s)$, $\psi(s)$, and $\varepsilon(s)$ by linear combinations of basic functions $e_1(s)$ through $e_k(s)$. Let $\boldsymbol{\alpha}$ be a vector consisting of coefficients corresponding to functions $\phi(s)$, $\theta(s)$, $\psi(s)$, and $\varepsilon(s)$. The potential energy, the work given by eq.(6), and the geometric constraints given by eqs.(7) and (8) are then functions with respect to vector $\boldsymbol{\alpha}$. Therefore, the deformed shape of a linear object can be derived by computing vector $\boldsymbol{\alpha}$. Namely, computation of the shape of the deformed object results in the minimization problem as follows:

$$\begin{array}{lll} \text{minimize} & : & U(\boldsymbol{\alpha}) - \displaystyle\sum_k \boldsymbol{p}_k \cdot \delta \boldsymbol{q}_k(\boldsymbol{\alpha}) \\ \text{subject to} & : & g_i(\boldsymbol{\alpha}) \leq 0, \quad i = 1, \cdots, u \\ & & h_j(\boldsymbol{\alpha}) = 0, \quad j = 1, \cdots, v \end{array} \tag{9}$$

Next, we convert the above minimization problem with constraints into the minimization problem without constraints by applying multiplier method[7]. The augmented objective function is then described as follows:

$$\begin{aligned} L_{t,r}(\boldsymbol{\alpha}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= U(\boldsymbol{\alpha}) - \sum_k \boldsymbol{p}_k \cdot \delta \boldsymbol{q}_k(\boldsymbol{\alpha}) \\ &+ \sum_{i=1}^{u} \frac{1}{2t_i} \left[\max\{0,\ \lambda_i + t_i g_i(\boldsymbol{\alpha})\}^2 - (\lambda_i)^2 \right] \\ &+ \sum_{j=1}^{v} \left[\mu_j h_j(\boldsymbol{\alpha}) + \frac{1}{2} r_j \{h_j(\boldsymbol{\alpha})\}^2 \right] \end{aligned} \tag{10}$$

where λ_i and μ_j are Kuhn-Tucker multipliers, and t_i and r_j are parameters for multiplier method. By minimizing this augmented objective function by use of an optimization algorithm such as quasi-Newton method, we can compute the shape of a deformed linear object numerically.

In eq.(10), $U(\boldsymbol{\alpha})$ has the dimension of energy while $g_i(\boldsymbol{\alpha})$ and $h_j(\boldsymbol{\alpha})$ have the dimension of distance and that of angle, respectively. So, it turns out that Kuhn-Tucker multipliers must have the dimension of force and that of moment, respectively. This implies that we can derive the force and the moment imposed on the object by solving this minimization problem. Reaction force and reaction moment due to geometric constraints $g_i(\boldsymbol{\alpha})$ and $h_j(\boldsymbol{\alpha})$ are given by the product of Kuhn-Tucker multipliers and the negative gradients of the constraints, that is , $-\lambda_i \nabla g_i(\boldsymbol{\alpha})$ and $-\mu_j \nabla h_j(\boldsymbol{\alpha})$, respectively[7]. From the above procedure, we can compute the deformed shape of a linear object and reaction wrenches acting on the object once a certain external wrench is specified. Therefore, we can derive allowable maximum external force $F_{\rm bfc}$ and allowable maximum external moment $M_{\rm bfc}$ by examining whether the bounded force closure condition given by eqs.(2) and (3) is satisfied or not at individual deformations.

5 Numerical Examples

In this section, we will demonstrate the effectiveness of our proposed approach with some numerical examples. In the first example, we will examine the relationship between inequality conditions and reaction forces. In the second example, we will compute the allowable maximum external force and moment to evaluate the stability of deformable object grasping. The following set of basic functions $e_1(s)$ through $e_{10}(s)$ are used in the computation of these examples:

$$\begin{aligned} & e_1(s) = 1, \quad e_2(s) = s, \\ & e_{2n+1}(s) = \sin \frac{2n\pi s}{L}, \\ & e_{2n+2}(s) = \cos \frac{2n\pi s}{L}. \quad (n = 1, 2, 3, 4) \end{aligned}$$

If a wrench is imposed on the object at point $P(s_{\rm ex})$, the distributed wrench along the central axis of the object changes discontinuously at this point. Thus, we will describe coefficient vector $\boldsymbol{\alpha}$ as follows:

$$\boldsymbol{\alpha} = \left\{ \begin{array}{ll} \boldsymbol{\alpha}_{\rm left} & (s \leq s_{\rm ex}) \\ \boldsymbol{\alpha}_{\rm right} & (s \geq s_{\rm ex}) \end{array} \right.$$

Note that the following equations must be satisfied at $s = s_{\rm ex}$:

$$[\,\phi,\ \ \theta,\ \ \psi,\ \ \varepsilon\,]^{\rm T}(\boldsymbol{\alpha}_{\rm left}) = [\,\phi,\ \ \theta,\ \ \psi,\ \ \varepsilon\,]^{\rm T}(\boldsymbol{\alpha}_{\rm right}).$$

Using the above equations, we will compute the deformed shape of a linear object and reaction forces imposed on it.

5.1 Inequality Conditions and Reaction Forces

The first example examines the relationship between inequality conditions and reaction forces. In this example, we assume that a linear object deforms along $z - x$ plane and the potential energy consists of flexural energy alone. Assume that the geometric

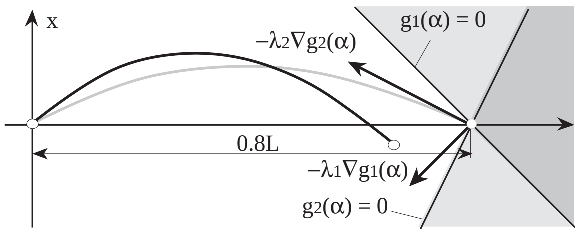


Figure 4: 2D-deformation of a linear object corresponding to inequality conditions

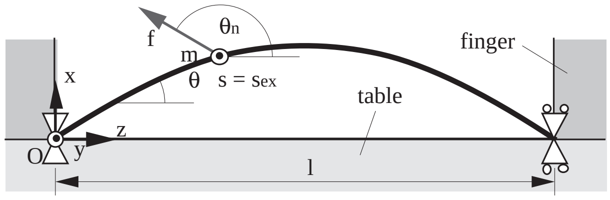


Figure 5: 2D-deformation of a linear object imposed an external force and moment

constraints imposed on the object are given by the following inequalities as illustrated in Figure 4.

$$g_1(\boldsymbol{\alpha}) = z(L) + x(L) - 0.8L \leq 0, \qquad (11)$$
$$g_2(\boldsymbol{\alpha}) = 2z(L) - x(L) - 1.6L \leq 0. \qquad (12)$$

Let $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ be reaction forces due to surfaces represented as $g_1(\boldsymbol{\alpha}) = 0$, and $g_2(\boldsymbol{\alpha}) = 0$, respectively. We find that $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are then computed as follows:

$$\boldsymbol{R}_1 = -\lambda_1 \nabla g_1(\boldsymbol{\alpha}) = [\,-3.67R_f/L, \;\; -3.67R_f/L\,]^{\mathrm{T}}$$
$$\boldsymbol{R}_2 = -\lambda_2 \nabla g_2(\boldsymbol{\alpha}) = [\,-7.33R_f/L, \;\; 3.66R_f/L\,]^{\mathrm{T}}$$

where R_f is the flexural rigidity. Resultant force is then represented as $\boldsymbol{R}_1 + \boldsymbol{R}_2 = [\,-11.0R_f/L, -0.01R_f/L\,]^{\mathrm{T}}$. Furthermore, it is found that the coordinates of the right endpoint of the object are equal to $[\,0, \;\; 0.8L\,]^{\mathrm{T}}$ from the computation. Let us assume that geometric constraints which the right endpoint is equal to $[\,0, \;\; 0.8L\,]^{\mathrm{T}}$ are imposed on the object. That is, the following equational constraints are imposed:

$$h_1(\boldsymbol{\alpha}) = z(L) - 0.8L = 0, \qquad (13)$$
$$h_2(\boldsymbol{\alpha}) = x(L) = 0. \qquad (14)$$

Then, it is found that the reaction force $\boldsymbol{R}$ in the above case is equal to the resultant force $\boldsymbol{R}_1 + \boldsymbol{R}_2$. Therefore, by use of our proposed procedure, reaction wrenches can be computed when the geometric constraints imposed on the object are represented as both equational and inequality conditions.

5.2 Allowable Maximum External Force and Moment

The second example shows the evaluation of the stability in deformable object grasping. Let us assume that the linear object deforms along $z-x$ plane and its potential energy U consists of flexural energy U_{flex} alone as well as the first example. The geometric constraints illustrated in Figure 5 are described as follows:

$$\dot{\theta}(0) = \dot{\theta}(L) = 0 \qquad (15)$$
$$x(L) = 0 \qquad (16)$$
$$z(L) - l \leq 0 \qquad (17)$$
$$-x(s) \leq 0, \;\; \forall s \in [\,0, \; L\,] \qquad (18)$$

Eq.(15) means that any moment cannot be exerted at both end points. Eqs.(16) and (17) describe that the right end point can move along z-axis. Eq.(18) shows that an arbitrary point of the object must not interfere with the table, which is defined by $x \leq 0$.

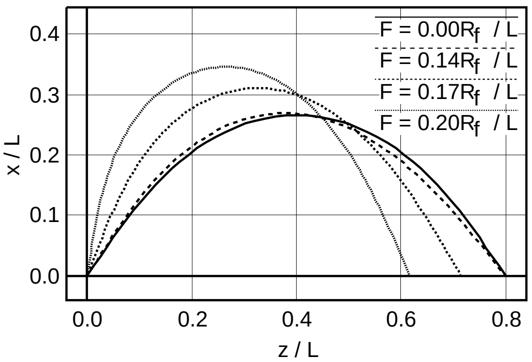
Let us exert an external force at a point corresponding to $s_{\mathrm{ex}} = 0.5L$ in the direction of $3\pi/4$ from the horizon. The deformed shapes are plotted in Figure 6-(a). From this figure, we find that the contact at the right endpoint is lost when the magnitude f exceeds $0.14R_f/L$. This shows that the maximum magnitude F in this case is equal to $0.14R_f/L$.

Let us exert an external moment around the y-axis at a point corresponding to $s_{\mathrm{ex}} = 0.5L$. The deformed shapes are plotted in Figure 6-(b). From this figure, we find that the contact at the right endpoint is lost when the magnitude m exceeds $0.17R_f$. This shows that the maximum magnitude M in this case is equal to $0.17R_f$.

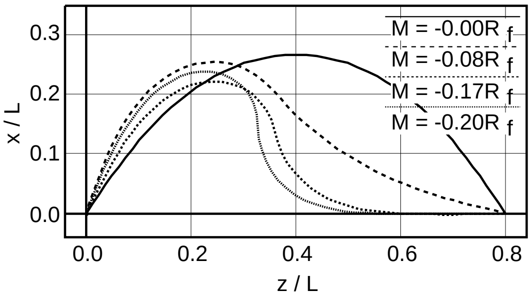
Allowable maximum external force As mentioned above, we can compute the maximum magnitude of external force and moment acting on a certain point of the object along a certain direction. First, we assume that the right end point of a linear object doesn't slip, namely, $x(L) = 0$. Next, we compute reaction forces F_x and F_z at the end point. If F_x/F_z is larger than the friction coefficient μ between the object and a finger, that point can slip actually. Therefore, the magnitude of external force/moment is maximum for stable grasping when reaction forces satisfy $F_x/F_z = \mu$. Computing the maximum magnitude corresponding to various acting point and direction, we can derive the allowable maximum external force F_{bfc} and the allowable maximum external moment M_{bfc}.

Figure 7 shows the relationship between the friction coefficient μ and the allowable maximum external force F_{bfc}. Upper bounds of forces have been computed for $s_{\mathrm{ex}} = 0.1aL$ $(a = 1, 2, \cdots, 9)$ and $\theta_n = \pm(4+b)\pi/8$ $(b = 0, 1, \cdots, 4)$. From this figure, it is found that F_{bfc} becomes larger when μ becomes larger or the distance between two end points of the object becomes smaller.

Allowable maximum external moment Figure 8 shows the relationship between the friction coefficient μ and the allowable maximum external moment M_{bfc}. Upper bounds of moments have been computed for



(a) external force



(b) external moment

Figure 6: Deformed shapes caused by external force and moment

$s_{\rm ex} = 0.1aL$ $(a = 1, 2, \cdots, 9)$. From this figure, it is also found that $M_{\rm bfc}$ becomes larger when μ becomes larger or the distance between two end points of the object becomes smaller.

From the above computation, it can be shown that the stability of grasping of a linear object depends upon the friction between the object and the fingers as well as the distance between the two endpoints.

6 Concluding Remarks

In this paper, we analyzed stable grasping of deformable objects based on the concept of bounded force closure. Firstly, we introduced the concept of bounded force closure, which is an extension of force closure. It was found that the allowable external force and moment was upper bounded in the grasping of deformable objects. Secondly, we investigated the necessary condition for bounded force closure and derived the properties of bounded force closure grasping. We found that a necessary condition for bounded force closure was given by a condition that the negative of the body force was involved in the interior set of reaction forces. Thirdly, we formulated the deformation of linear objects as an example of deformable objects. We proposed a procedure to evaluate stability in deformable object grasping. Finally, some numerical examples were shown in order to demonstrate the effectiveness of our proposed method. It was shown that the stability of deformable object grasping depended upon the friction and the shape at the grasping.

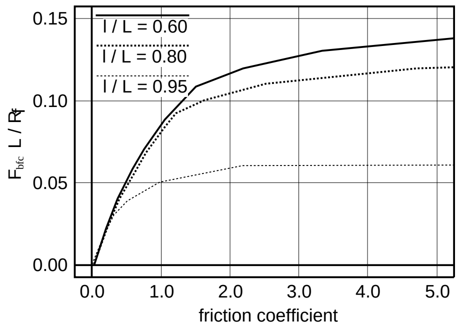


Figure 7: Allowable maximum external force

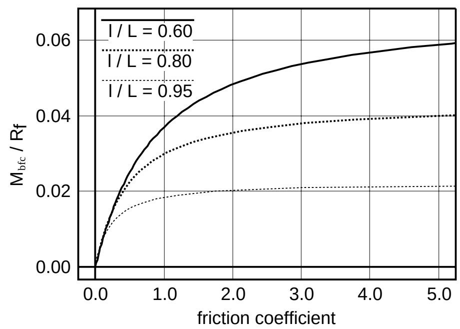


Figure 8: Allowable maximum external moment

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