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Modeling of Hysteresis in Deformation of Rodlike Objects Toward Their Manipulation

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Abstract

A systematic approach to the modeling of deformable rodlike objects is presented. Various rodlike objects such as cords and wires are manipulated in many manufacturing processes. In such processes, it is important for successful manipulation to evaluate their shapes on a computer in advance because their shapes can be changed easily and their deformation often shows hysteresis properties. In this paper, we will develop an analytical method to model the shape of deformable rodlike objects including hysteresis properties. First, we will investigate the mechanism of hysteresis. Second, the potential energy of a rodlike object and the geometric constraints imposed on it are formulated. The shape of the object can be derived by minimizing the potential energy under the geometric constraints. Thirdly, a procedure to compute the shape of a deformed rodlike object is developed by applying a nonlinear programming technique. Finally, we show some numerical examples with hysteresis using our proposed method.

1 Introduction

Various deformable objects including cords and wires are manipulated in many manufacturing processes. Deformation of these objects is often utilized in order to manipulate them successfully while the manipulation sometimes fails because of unexpected deformation of them. Modeling of deformable objects is thus required so that the shape of the objects can be evaluated on a computer in advance. Especially, evaluating the shape of rodlike objects is important because their shape can be changed easily by small forces/moments which are imposed on them.

There are many studies about the modeling and manipulation of rodlike objects such as flexible beams or wires. Zheng et al derived strategies to the insert a flexible beam into a hole without wedging or jamming[1]. We have developed a modeling technique of rodlike objects such as wires considering its static deformation[2]. Nakagaki et al have studied insertion task of a flexible wire into hole using the wire model and visual tracking[3]. Wada et al have analyzed the deformation of knitted fabrics using string model[4]. Nishinari has developed string model and analyzed loop structure of the string[5]. The paper treated the deformation in a two-dimensional plane and it is impossible to describe the twist of the objects. However, loop structure occurs essentially by the three-dimensional structure of the string objects and their twist in three-dimensional space.

Experimental manipulation of flexible, rod-like objects has resulted in a hysteresis of the shape that the object assumes. The history of the deformation is important because the shape of the object has been observed to depend on the history. Namely, the shape may change according to the sequence of operations. Thus, this hysteresis property must be investigated so that the deformation during a series of operations can be evaluated on a computer.

In this paper, we will develop an analytical method to model the shape of deformable rodlike objects such as cords and wires that is capable of describing hysteresis property.

First, we will investigate mechanism of hysteresis in deformation processes based on the potential energy of deformable objects. Then, we will point out the importance of local minimum of the energy. Second, a geometric representation to describe the shape of a rodlike object with bending and torsional deformation is introduced. The potential energy of the object and the geometric constraints imposed on it are then formulated. The shape of the object in the stable state can be derived by minimizing the potential energy under the geometric constraints. Thirdly, a procedure to compute the shape of a deformed rodlike object is developed by applying a nonlinear programming technique. Finally, numerical examples of the shape transition with hysteresis are shown. As the result, we conclude that our proposed modeling method accurately describes the deformation including hysteresis.

2 Hysteresis in Deformation of Rodlike Object

2.1 Experiments

First, we will investigate the hysteresis of rodlike object deformation experimentally. We experiment with a metal wire, whose flexural and torsional rigidity are 6.6×10^{-4}[N$\cdot m^2$] and 2.3×10^{-4}[N$\cdot m^2$], respectively, in order to demonstrate that the shape of a rodlike object depends on the history of manipulative operation. Fig.1 illustrates the experimental setup for this purpose. Two robot hands can control the position and the orientation of both endpoints of a wire. In the initial state, one endpoint is rotated by ω_0[rad] with keeping the wire straight. The distance between two endpoints is then decreased by controlling the two robot hands. We measure the shape of the deformed

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wire with two cameras. Next, the distance is increased and the shape of the deformed wire is measured.

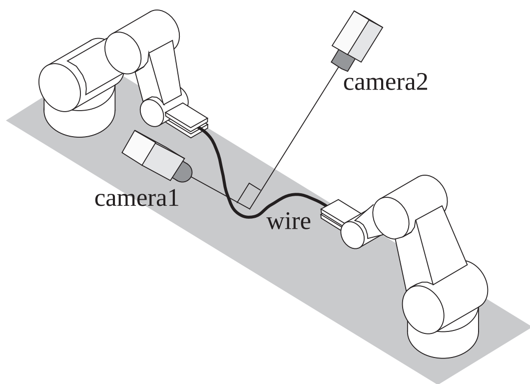


Figure 1: Sketch of devices for experiment

Fig.2 shows experimental results with the hysteresis property. The initial torsional angle ω_0 is equal to 2.25π. When ω_0 is less than 2π, this hysteresis property does not appear.

2.2 Hypothesis on Mechanism of Hysteresis

A deformable object has a stable shape such that its energy reaches of the minimum value in static deformation. Note that there exist multiple shapes with the same constraints. Thus, we can find that the shape can fall into a local minimum. These local minima play an important role for hysteresis property. Namely, the resultant shape from a current shape when the constraints are changed depends on the current shape. We regard this as the main reason for hysteresis. Fig.3 illustrates a schematic image of hysteresis mechanism. Suppose that there exist two minima shapes in the admissible region with certain constraints. If the previous shape is in shape(a), the next shape will be in shape(b), while if in shape(c) then in shape(d), even though the global minimum occurs in shape(d).

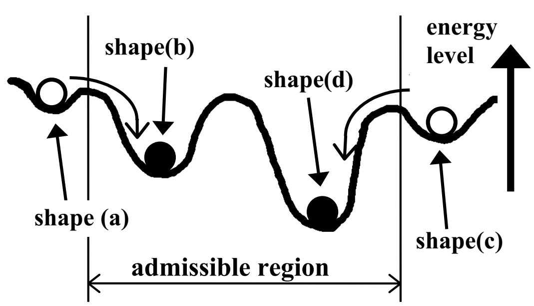


Figure 3: Schematic image of hysteresis mechanism

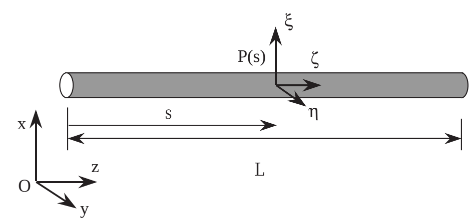
3 Modeling of Rodlike Object Deformation

3.1 Geometric Representation

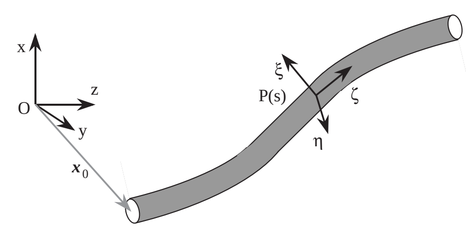
In this section, we will formulate the geometrical shape of a rodlike object in three-dimensional space. Let L be the length of the object along its central axis and s be the distance from one endpoint of the object along its central axis. In order to describe the deformation of a rodlike object, we will introduce the global space coordinate system and the local object coordinate systems at individual points on the object, as shown in Fig.4. Let $O-xyz$ be the coordinate system fixed in space. Let $P-\xi\eta\zeta$ be the coordinate system fixed on an arbitrary point $P(s)$ of the object and $\boldsymbol{x}(s) = [\, x(s),\ y(s),\ z(s)\,]^{\mathrm{T}}$ be the spatial coordinates corresponding to point $P(s)$. Select the direction of the local coordinate system $P-\xi\eta\zeta$ so that ζ-axis is aligned with the central axis of the object. In addition, select the direction of coordinates so that ξ-axis, η-axis, and ζ-axis are parallel to x-axis, y-axis, and z-axis, respectively, in the undeformed shape. Deformation of the rodlike object is then given by the relationship between the local coordinate at each point and the global coordinates. We describe the orientation of the local coordinate system by use of Eulerian angles $\phi(s)$, $\theta(s)$, and $\psi(s)$. The rotational transformation from $P-\xi\eta\zeta$ to $O-xyz$ is expressed by the following rotational matrix.

$$\begin{bmatrix} C_\theta C_\phi C_\psi - S_\phi S_\psi & -C_\phi C_\theta S_\psi - S_\phi C_\psi & C_\phi S_\theta \\ S_\phi C_\theta C_\psi + C_\phi S_\psi & -S_\phi C_\theta S_\psi + C_\phi C_\psi & S_\phi S_\theta \\ -S_\theta C_\psi & S_\theta S_\psi & C_\theta \end{bmatrix} \tag{1}$$

For the sake of simplicity, $\cos\theta$ and $\sin\theta$ are abbreviated as C_θ and S_θ, respectively.



(a) natural shape



(b) deformed shape

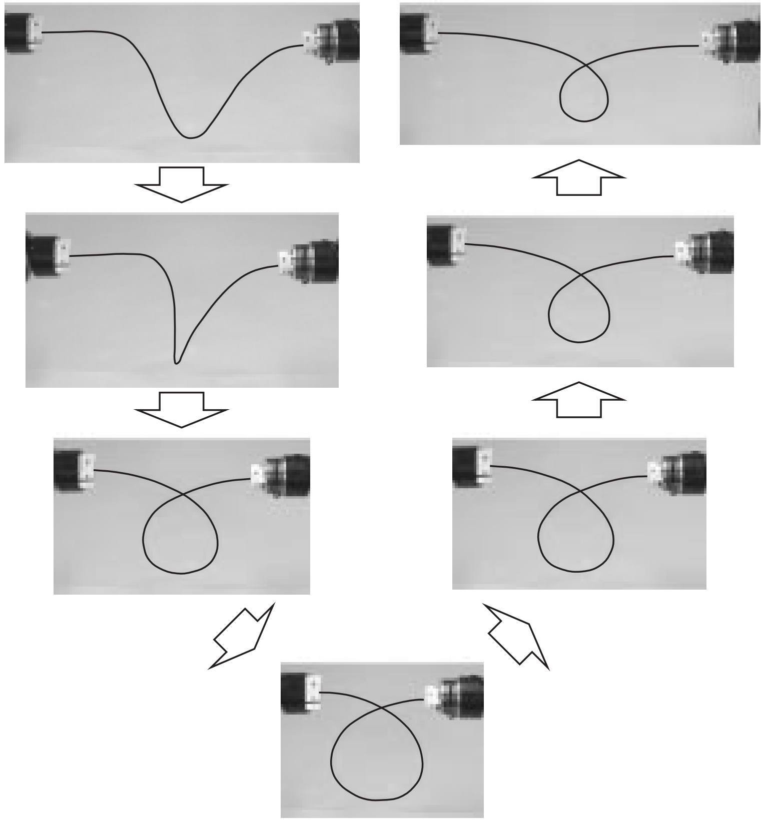
Figure 4: Coordinates systems describing object deformation

Let $\boldsymbol{\xi}(s)$, $\boldsymbol{\eta}(s)$, and $\boldsymbol{\zeta}(s)$ be unit vectors along ξ-axis, η-axis, and ζ-axis, respectively, at point $P(s)$ on the deformed shape. Vectors $\boldsymbol{\xi}(s)$, $\boldsymbol{\eta}(s)$, and $\boldsymbol{\zeta}(s)$ coincide to the first column, the second column, and the third column of the above rotational matrix, respectively. Note that $\boldsymbol{\zeta}(s)$ is the unit tangential vector at point $P(s)$. Namely, vector $\boldsymbol{\zeta}(s)$ is equal to the derivative $d\boldsymbol{x}/ds$. Then, the spatial coordinates can be computed as follows:

$$\boldsymbol{x}(s) = \boldsymbol{x}_0 + \int_0^s \boldsymbol{\zeta}(s)ds \tag{2}$$

where $\boldsymbol{x}_0 = [\, x_0,\ y_0,\ z_0\,]^{\mathrm{T}}$ denotes spatial coordinates at the end point corresponding to $s = 0$.

In order to express bending and torsional deformation of the object, let us describe the curvature of the object and its torsional ratio. Recall that vector $\boldsymbol{\zeta}(s)$ is the unit tangential vector at point $P(s)$. Then bend at point $P(s)$ is thus given by the included angle between $\boldsymbol{\zeta}(s)$ and $\boldsymbol{\zeta}(s+ds)$. Thus, the bend of a rodlike object at point $P(s)$ is given by



(a) shortening the distance (b) lengthening the distance

Figure 2: Experimental results with hysteresis in deformation ($\omega_0 = 2.25\pi$)

$$\kappa^2 = ||\frac{d\boldsymbol{\zeta}(s)}{ds}||^2 = \left(\frac{d\theta}{ds}\right)^2 + \sin^2\theta\left(\frac{d\phi}{ds}\right)^2 \quad (3)$$

The twist at point $P(s)$ is defined as the torsional angle around ζ-axis, which causes the difference between $\boldsymbol{\xi}(s)$ and $\boldsymbol{\xi}(s+ds)$. Thus, the twist of a rodlike object at point $P(s)$ is thus given by

$$\omega^2 = ||\frac{d\boldsymbol{\xi}(s)}{ds} \times \boldsymbol{\zeta}(s)||^2 = \left(\frac{d\phi}{ds}\cos\theta + \frac{d\psi}{ds}\right)^2. \quad (4)$$

From the above discussion, we find that the geometrical shape of a deformed rodlike object can be represented by three variables, that is, Eulerian angles ϕ, θ, and ψ. Note that each variable depends upon parameter s.

3.2 Formulation of Potential Energy

Let us formulate the potential energy of a rodlike object. Applying Bernoulli and Navier's assumption, it turns out that the potential energy U is described as follows:

$$U = \frac{1}{2}\int_0^L R_f \kappa^2 ds + \frac{1}{2}\int_0^L R_t \omega^2 ds + \int_0^L D x ds \quad (5)$$

where R_f, R_t, and D represent the flexural rigidity, the torsional rigidity, and the weight per unit length at point $P(s)$, respectively. Note that R_f, R_t, and D may vary with respect to variable s in general.

3.3 Formulation of Geometric Constraints

Consider a constraint that specifies the positional relationship between two points on the object. Let $\boldsymbol{l} = [\ l_x,\ l_y,\ l_z\ ]^{\mathrm{T}}$ be a predetermined vector describing the relative position between two operational points, $P(s_a)$ and $P(s_b)$. Recall that the spatial coordinates corresponding to parameter s is given by eq.(2). Thus, the following condition must be satisfied:

$$\boldsymbol{x}(s_b) - \boldsymbol{x}(s_a) = \boldsymbol{l} \quad (6)$$

The orientation at some points of the object must also be controlled during the operation. These orientational constraints are simply described as follows:

$$\begin{aligned} \phi(s_c) &= \phi_c \\ \theta(s_c) &= \theta_c \\ \psi(s_c) &= \psi_c \end{aligned} \quad (7)$$

where ϕ_c, θ_c, and ψ_c are predefined angles at one operational point $P(s_c)$.

Note that any points on a rodlike object must be located outside each obstacle or on it when the contact between the object and rigid obstacles yields. Let us describe the surface of an obstacle fixed on space by function $h(x, y, z) = 0$. Suppose that value of the function is positive inside the obstacle and is negative outside it. The condition that a rodlike object is not interfered with this obstacle is then described as follows:

$$h(\boldsymbol{x}(s)) \leq 0, \quad \forall s \in [0, L] \tag{8}$$

where $\boldsymbol{x}(s)$ is described in eq.(2). Note that condition that an object is not interfered with obstacles is described by a set of inequalities, since mechanical contacts between the objects constraints the object motion unidirectionally.

In order to avoid the interference with itself, a rodlike object must satisfy the following conditions.

$$\begin{gathered} |\boldsymbol{x}(s_i) - \boldsymbol{x}(s_j)| \geq 2r, \\ \forall s_i, s_j \in [0, L], \ \text{s.t.} \ |s_i - s_j| \geq 2r \end{gathered} \tag{9}$$

where r represents the radius of a rodlike object.

[**Conservation of Rotation**]

Let us consider a case that an end point of the object is twisted 2π, and the distance between both ends reduced by maintaining the orientations of both ends, as shown in experimental results in **Section2**. In such cases, the rotation imposed on the object is conserved. Namely, the object has same quantity of rotation in it even the case that a part of the twist is changed into bending. Here, we introduce a method to describe quantity of rotation in the object using infinitesimal rotation of the object with respect to each axis of body fixed frame. Namely, we define infinitesimal rotation vector $\boldsymbol{dr}(s)/ds$ as follows:

$$\begin{aligned} &\frac{\boldsymbol{dr}(s)}{ds} \triangleq [dr_\xi/ds \ , \ dr_\eta/ds \ , \ dr_\zeta/ds]^T \\ &= \begin{bmatrix} \frac{d\theta(s)}{ds}\sin(\psi(s)) - \frac{d\phi(s)}{ds}\sin(\theta(s))\cos(\psi(s)) \\ \frac{d\theta(s)}{ds}\cos(\psi(s)) + \frac{d\phi(s)}{ds}\sin(\theta(s))\sin(\psi(s)) \\ \frac{\phi(s)}{ds}\cos(\theta(s)) + \frac{\psi(s)}{ds} \end{bmatrix} \end{aligned} \tag{10}$$

In eq.(10), dr_ξ/ds, dr_η/ds, and dr_ζ/ds denote the rotation of the object around ξ, η, and ζ-axis in the body fixed frame, respectively. Furthermore, we define the total rotation through the object ($s \in [0, L]$) as follows:

$$\boldsymbol{f} = \int_0^L \frac{\boldsymbol{dr}(s)}{ds} ds \tag{11}$$

As result, we can describe the rotation of the object by Euclidian norm of the vector $\boldsymbol{f}$, $||\boldsymbol{f}||$. Thus, we can utilize the following equation as a constraint:

$$||\boldsymbol{f}|| = |\rho|, \tag{12}$$

where ρ [rad] denotes the initial twist angle exerted on the object.

On the other hand, we cannot treat the sign of the rotation with eq.(12). The sign of the rotation can be described by that of the rotation around ζ-axis. Namely, we can describe the sign by eq.(13).

$$\text{sgn}(\rho) = \text{sgn}(\int_0^L \boldsymbol{dr}_\zeta(s)/ds \ ds). \tag{13}$$

From the above discussions, the following constraints are derived:

$$\phi(0) = \theta(0) = \psi(0) = 0 \tag{14}$$

$$\boldsymbol{\xi}(L) = [1, 0, 0]^T, \ \ \boldsymbol{\eta}(L) = [0, 1, 0]^T \tag{15}$$

$$||\boldsymbol{f}|| = |\rho| \tag{16}$$

$$\text{sgn}(\int_0^L \boldsymbol{dr}_\zeta(s)/ds \ ds) = \text{sgn}(\rho) \tag{17}$$

Eqs.(16), (17) are constraints that ensure that the initial twist put into the system remains in the system. Eq.(15) denotes constraints that ensure that the orientations of the local coordinate axes at $s = L$ coincide with space coordinate axes.

From the above discussion, the shape of a rodlike object is determined by minimizing potential energy described in eq.(5) under the geometric constraints represented by eqs.(6) – (9), or (14) – (17). Namely, computation of the shape of a deformed object results in a variational problem under equality and inequality conditions.

4 Computation of Deformed Shape

4.1 Computation Procedure

Computation of the shape of a rodlike object results in a variational problem as mentioned in the previous section. In this paper, we will develop a direct method based on Ritz's method [6] and a nonlinear programming technique because the variational problem in this case includes inequality conditions.

Let us express functions $\phi(s)$, $\theta(s)$, and $\psi(s)$ by linear combinations of basic functions $e_1(s)$ through $e_n(s)$:

$$\begin{bmatrix} \phi(s) \\ \theta(s) \\ \psi(s) \end{bmatrix} \triangleq \begin{bmatrix} \boldsymbol{a}_\phi{}^{\mathrm{T}} \\ \boldsymbol{a}_\theta{}^{\mathrm{T}} \\ \boldsymbol{a}_\psi{}^{\mathrm{T}} \end{bmatrix} \cdot \boldsymbol{e}(s) \tag{18}$$

where $\boldsymbol{a}_\phi$, $\boldsymbol{a}_\theta$, and $\boldsymbol{a}_\psi$ are vectors consisting of coefficients corresponding to functions $\phi(s)$, $\theta(s)$, and $\psi(s)$, respectively, and vector $\boldsymbol{e}(s)$ is composed of basic functions $e_1(s)$ through $e_n(s)$. Let us describe the whole coefficient vector $\boldsymbol{a} = [\ \boldsymbol{a}_\phi{}^{\mathrm{T}}, \ \boldsymbol{a}_\theta{}^{\mathrm{T}}, \ \boldsymbol{a}_\psi{}^{\mathrm{T}}\ ]^{\mathrm{T}}$. Substituting eq.(18) into eq.(5), potential energy U is described by a function of coefficient vector; $\boldsymbol{a}$. The geometric constraints are also described by conditions involving the coefficient vector. As a result, a set of the geometric constraints is expressed with respect to the coefficient vectors.

The shape of the object can be then derived by computing the coefficient vector $\boldsymbol{a}$ that minimizes the potential energy under the geometric constraints. This minimization problem under equality and inequality conditions can be solved by use of a nonlinear programming technique such as multiplier method [7]. The shape of the object corresponding to the coefficient vector can be computed by use of eq.(2).

4.2 Algorithm for Minimizing Potential Energy

As mentioned in **Section 2.2**, it is important what kind of optimizing method is utilized for minimizing the potential energy. It should be noted that the optimization techniques have developed such that they intend to obtain a global minimum value of a given function. In other words, they attempt to avoid local minima. Thus, such algorithms are not appropriate for our purpose since local minimums are important for describing object deformations, especially hysteresis. Therefore, we have to develop a new algorithm suitable for physical meaning of object deformation. This is one of the most important future works.

For simplicity, we employ Nelder-Mead Simplex Method for minimizing potential energy in this paper since this method can obtain local minimum value relatively easier in the case that a range of search small.

5 Numerical Example

In this section, we will show some numerical examples using our proposed approach in order to investigate the validity of the modeling method and the idea of hysteresis mechanism.

The following set of basic functions $e_1(s)$ through $e_{10}(s)$ are used in the computation of these examples:

$$e_1(s) = 1, \quad e_2(s) = s,$$
$$e_{2n+1}(s) = \sin \frac{n\pi s}{L},$$
$$e_{2n+2}(s) = \cos \frac{n\pi s}{L}. \quad (n = 1, 2, 3, 4)$$

We fix one endpoint of a rodlike object and rotate the other 2π[rad] around the central axis. The orientations of both ends are fixed in all the below processes. Then, the distance between two ends is shortened. After that, the distance is lengthened to the initial state.

The constraints of the orientation can be described by eqs. (14), (15). The constraints of 2π initial rotation are described by eqs. (16) and (17). The constraints of the distance between two ends can be described as follows:

$$\boldsymbol{x}(L) - \boldsymbol{x}(0) = [0, 0, l_z]^T, \tag{19}$$

where l_z denotes the distance between two ends. In the simulation, l_z is varied in order to describe shortening and lengthen processes.

Fig.6 shows computational results. In this figure, circles denote positions of $P(s)$, bars drawn from the circles are ξ-axis at $P(s)$. Qualitatively speaking, we can find that the simulation results are similar as the experimental results in Fig.2. In the beginning of shortening process, the object exhibits bending and torsional deformations. In end of the shortening, the objects make knot structure. On the other hand, the knot structure is maintained a little before the end of the process in lengthening processes. Finally, from just before the end of the process, the knot disappears.

Fig.5 shows the energy levels during the deformation. From this figure, we can find that there are different energy levels between shortening and lengthening processes.

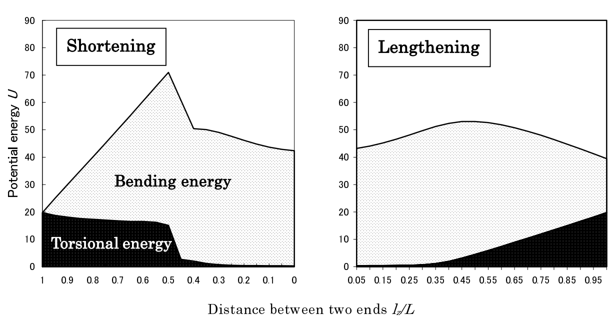


Figure 5: Energy levels during deformation

As shown in simulation results, the hysteresis property is described with the proposed method as well as the bending and torsional deformations. Thus, we can finally conclude that our proposed method is appropriate to describe deformed shapes of rodlike objects including hysteresis.

6 Conclusions

In this paper, we developed an analytical method to model the shape of deformable rodlike objects including hysteresis property. First, we have proposed a hypothesis on the mechanism of hysteresis in deformation of deformable object based on their potential energy. In this consideration, we emphasize the importance of local minima of the energy. Second, a geometric representation to describe the shape of a rodlike object was introduced. The potential energy of the object and the geometric constraints imposed on it were formulated using Eulerian angles. The shape of the object in the stable state was derived by minimizing the potential energy under the geometric constraints. There, we derived the method to describe conservation of the rotation imposed on the object. Thirdly, the procedure to compute the shape of a deformed rodlike object was developed by applying a nonlinear programming technique. We also proposed to utilize the technique that is suitable for falling into local minimums. Finally, numerical examples were shown in order to investigate the validity of our proposed modeling method and the idea of hysteresis mechanism. As a result, we conclude that our proposed method accurately describes deformed shapes of rodlike objects including hysteresis.

It is expected that this approach enables us to plan manipulative operations of rodlike objects without hysteresis properties or to utilize such properties for their manipulation.

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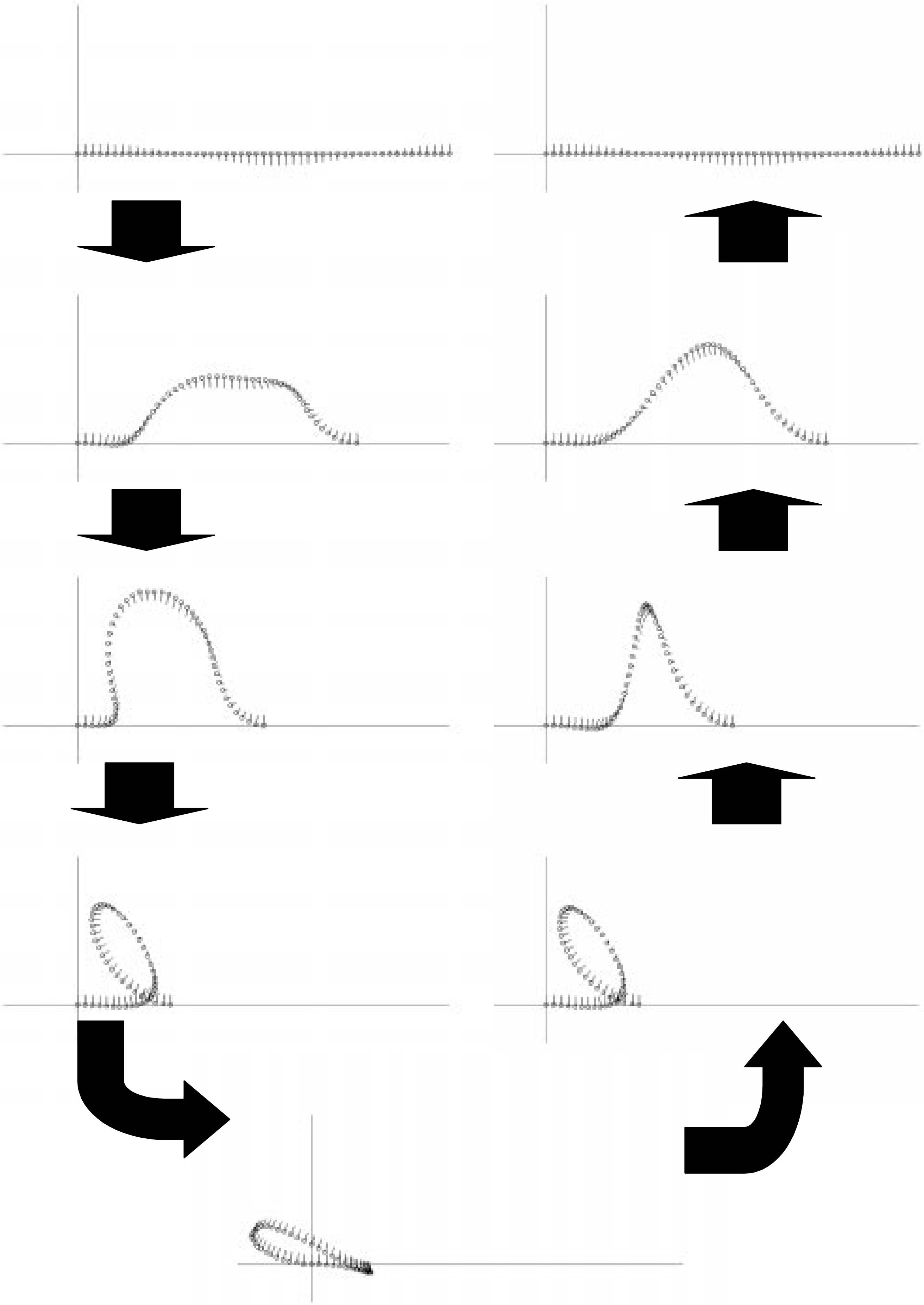

Figure 6: Computational results with hysteresis