Dynamic Analysis of Rodlike Object Deformation towards Their Dynamic Manipulation

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Abstract

A dynamic motion analysis of deformable rodlike objects is presented. In manufacturing processes, there are many manipulative operations which deal with deformable objects. Evaluation of the shapes of these objects is important for their manipulative operations because their deformation can cause both success of such operations if it is utilized effectively and their failure if the deformation is unexpected. Furthermore, if deformable objects are operated quickly, the dynamical effect of them cannot be neglected when we evaluate their shapes. Thus, both static and dynamic analysis of objects deformation is required so that the shape of the objects can be evaluated on a computer in advance. In this paper, we will analyse the dynamic deformation of deformable rodlike objects. First, a geometric representation to describe the shape of a rodlike object with dynamic deformation is introduced. The potential and the kinetic energy of the object and the geometric constraints imposed on it are then formu-lated. The shape of the dynamically deforming object can be derived by minimizing the difference between the kinetic energy and the potential energy under the geometric constraints. Next, a procedure to compute the deformed shape is developed by use of Euler's approach. Finally, some numerical examples are shown in order to demonstrate how the proposed approach computes the shapes of deformed rodlike objects.

1 Introduction

In manufacturing, the automation of handling and manipulative processes which deal with deformable objects such as rubber tubes, sheet metals, cords, leather products, and paper sheets has been done but it is not enough to satisfy our requests. In manipulative processes, if the shape of deformed objects can be utilized actively, we can operate them successfully. If it is unexpected, however, the operations may result in failure. Modeling of deformable objects is thus necessary in order to evaluate the shape of them on a computer in advance and to derive task strategies to carry out manipulative operations successfully by avoiding



(a) slow manipulation (b) quick manipulation

Figure 1: Example of manipulation utilizing dynamic deformation of rodlike object

deformation or by utilizing it. Zheng et al derived strategies to insert a flexible beam into a hole without wedging or jamming[1]. Villarreal et al developed strategies by use of the buffer zone, which represents the compliance of flexible parts[2]. We have developed a modeling technique of rodlike objects such as wires considering its static deformation[3]. Nakagaki et al have been studying insertion task of a flexible wire into hole[4]. However, these studies do not consider the dynamical effect of deformable objects when they deform. Modeling of dynamic deformation becomes important because the dynamical effect of them cannot be neglected if they are operated quickly by humans or machines. Furthermore, by considering dynamic deformation, we can derive new task strategies which cannot be derived when only static deformation is considered. For example, when we manipulate a rodlike object slowly as shown in Figure 1-(a), it will impact against an obstacle. But, quick manipulation can avoid impacting as shown in Figure 1-(b) even if a manipulator tracks the same trajectory. Also, we can identify whipping or lariating as one of good strate-gies to operate the far end of a deformable rodlike object such as a whip or a lariat quickly. Therefore, it is important for quick manipulation of deformable objects to evaluate the shape of them which deform



Figure 2: Motion and deformation of rodlike object

dynamically in advance.

In this paper, we will analyse the dynamic deformation of deformable rodlike objects. First, a geometric representation of dynamic deformation of a rodlike object in 3-dimensional space is established. Secondly, the potential and the kinetic energy of the object and the geometric constraints imposed on it are formulated. Thirdly, procedure to compute the deformed shape is developed by use of Euler's approach. Finally, some numerical examples are shown in order to demonstrate how the shapes of deformed rodlike objects are computed using the proposed approach.

2 Modeling of Rodlike Object Deformation

2.1 Geometric Representation of Deformed Rodlike Objects

In this section, we will formulate the geometrical shape of a rodlike object, which moves and deforms dynamically in three-dimensional space. Let L be the length of the object, s be the distance from one endpoint of the object along it, and t be the time. Let us introduce the global space coordinate system and the local object coordinate systems at individual points on the object and at each time, as shown in Figure 2, in order to describe the motion and the deformation of a rodlike object.

Let O-xyz be the coordinate system fixed on space and $P(s,t) - \xi \eta \zeta$ be the coordinate system fixed on an arbitrary point of the object at distance s and time t. Select the direction of the coordinates so that the ξ -axis, η -axis, and ζ -axis are parallel to x-axis, yaxis, and z-axis, respectively, in natural state where the object neither move nor deform. Then, the motion and the deformation of the object are represented by the relationship between the local coordinate system $P(s,t) - \xi \eta \zeta$ and the global coordinate system O-xyz. Let us describe the orientation of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(s,t)$, $\theta(s,t)$, and $\psi(s,t)$. Namely, rotational transformation from coordinate system $\mathbf{P}(s,t) - \xi \eta \zeta$ to coordinate system $\mathbf{O}-xyz$ is expressed by the following rotational matrix:

$$\begin{bmatrix} C_{\theta}C_{\phi}C_{\psi} - S_{\phi}S_{\psi} & C_{\theta}S_{\phi}C_{\psi} + C_{\phi}S_{\psi} & -S_{\theta}C_{\psi} \\ -C_{\theta}C_{\phi}S_{\psi} - S_{\phi}C_{\psi} & -C_{\theta}S_{\phi}S_{\psi} + C_{\phi}C_{\psi} & S_{\theta}S_{\psi} \\ S_{\theta}C_{\phi} & S_{\theta}S_{\phi} & C_{\theta} \end{bmatrix}$$

For the sake of simplicity, $\cos \theta$ and $\sin \theta$ are abbreviated as C_{θ} and S_{θ} , respectively. Note that the Eulerian angles depend upon parameters s and t.

By using above rotational matrix, a unit vector along ζ -axis at the natural state is transformed into the following vector due to the object motion and deformation:

$$\boldsymbol{\zeta}(s,t) = \begin{bmatrix} \cos\phi\sin\theta\\ \sin\phi\sin\theta\\ \cos\theta \end{bmatrix}.$$
(1)

Let $\boldsymbol{x} = [x(s,t) \ y(s,t) \ z(s,t)]^{\mathrm{T}}$ be spatial coordinates corresponding to point $\mathrm{P}(s,t)$ along x-, y-, and z-axis, respectively. The spatial coordinates can be computed by integrating the above vector. Namely,

$$\boldsymbol{x} = \boldsymbol{x}_0 + \int_0^s \boldsymbol{\zeta} \, ds \tag{2}$$

where \boldsymbol{x}_0 denotes the coordinate at the end point corresponding to s = 0, which is represented as a function of time t.

Let us describe the curvature of the object and its torsional ratio at time t in order to express bending and torsional deformations of the object. Let ϕ_0 , θ_0 , and ψ_0 be Eulerian angles at the end point corresponding to s = 0. Then, these are represented as a function of time t. Let $\kappa(s,t)$ and $\chi(s,t)$ be the curvature and the torsional ratio at point P(s,t), respectively. The curvature and the torsional ratio can be described by use of Eulerian angles ϕ , θ , ψ , and θ_0 as follows:

$$\begin{aligned} \kappa^2 &= \left(\frac{\partial\theta}{\partial s}\right)^2 + \left(\frac{\partial\phi}{\partial s}\right)^2 \sin^2(\theta - \theta_0),\\ \chi^2 &= \left\{\frac{\partial\psi}{\partial s} + \frac{\partial\phi}{\partial s}\cos(\theta - \theta_0)\right\}^2. \end{aligned}$$

Next, let us describe the velocity and the angular velocity of the object at time t, in order to express motion of the object. Let v be the velocity of the object at the point P(s,t), namely,

$$\boldsymbol{v} = \frac{\partial \boldsymbol{x}}{\partial t}.$$

Furthermore, let $\omega_1(s,t)$ and $\omega_2(s,t)$ be the angular velocity for deformation around the axis which intersects with the central axis perpendicularly, and that around the central axis at point P(s,t), respectively as shown in Figure 3. It is found that these two angular velocities can be described by use of Eulerian angles $\phi, \theta, \psi, \phi_0, \theta_0$, and ψ_0 as follows:

$$\omega_1^2 = \left\{ \frac{\partial(\theta - \theta_0)}{\partial t} \right\}^2 + \left\{ \frac{\partial(\phi - \phi_0)}{\partial t} \right\}^2 \sin^2(\theta - \theta_0),$$
$$\omega_2^2 = \left\{ \frac{\partial(\psi - \psi_0)}{\partial t} + \frac{\partial(\phi - \phi_0)}{\partial t} \cos(\theta - \theta_0) \right\}^2.$$

From the above discussion, the geometric shape of a moving and deforming rodlike object can be represented by both three variables, ϕ , θ , and ψ , which depend upon distance s and time t, and three variables, ϕ_0 , θ_0 , and ψ_0 , which depend upon time t alone.



Figure 3: Angular velocities of rodlike object

2.2 Potential Energy, Kinetic Energy, and Geometric Constraints

In this paper, we will adopt the Hamilton's principle that the time integral of the difference between the kinetic energy of the object and its potential energy reaches the minimum when the object moves dynamically. In this section, we will formulate the potential energy and the kinetic energy of a rodlike object and geometric constraints imposed on it.

First, let us formulate the potential energy of a rodlike object. Applying Bernoulli and Navier's assumption, we can describe the potential energy U at time tas follows:

$$U = U_{\text{flex}} + U_{\text{tor}}$$

where U_{flex} and U_{tor} represent flexural and torsional energy, respectively.

Flexural energy U_{flex} and torsional energy U_{tor} of the object can be computed by integrating flexural energy and torsional energy at point P(s) over the object. Assuming that the curvature and the torsional angle are proportional to bending moment and twisting moment at each point P(s,t), respectively, it turns out that the potential energy U(t) can be described as follows:

$$U(t) = \int_0^L \frac{R_f}{2} \kappa^2 \, ds + \int_0^L \frac{R_t}{2} \chi^2 \, ds \tag{3}$$

where R_f and R_t represent the flexural rigidity and the torsional rigidity at point P(s), respectively.

Next, let us formulate the kinetic energy of a rodlike object. Considering translation, rotation as a rigid body, and rotational deformation of the object, the kinetic energy T at time t is described as follows:

$$T(t) = T_{\text{trans}} + T_{\text{rot}} + T_{\text{def}}$$

where T_{trans} , T_{rot} , and T_{def} represent the kinetic energy for translation, that for rotation as a rigid body, and that for deformation, respectively. They can also be computed by integrating these kinetic energy at point P(s,t) over the object. Namely,

$$T(t) = \int_{0}^{L} \frac{\rho}{2} v^{2} ds + \int_{0}^{L} \frac{\rho}{2} f(x, \phi_{0}, \theta_{0}, \psi_{0}) ds + \int_{0}^{L} \frac{I_{1}}{2} \omega_{1}^{2} ds + \int_{0}^{L} \frac{I_{2}}{2} \omega_{2}^{2} ds$$
(4)

where ρ , I_1 , and I_2 represent the mass of the object per unit length, the moment of inertia around the axis which intersects with the central axis of the object perpendicularly, and that around the central axis of it, respectively. Function $f(\boldsymbol{x}, \phi_0, \theta_0, \psi_0)$ can be determined as rotational energy of a rigid object.

By applying Hamilton's principle, the following functional should reach to its minimum at the actual motion and deformation:

$$F = \int_{t_1}^{t_2} (T - U) \, dt. \tag{5}$$

Some geometric constraints are imposed on the object because a rodlike object may interact with other objects such as fingertips and obstacles. Let us derive the geometric constraints imposed on the object. The relative position between two points on the object is often controlled during object operations. Consider a constraint that specifies the positional relationship between two points on the object through t_1 to t_2 . Let $\boldsymbol{l} = [l_x \ l_y \ l_z]^{\mathrm{T}}$ be a predetermined vector describing the relative position between two operational points, $P(s_1)$ and $P(s_2)$. Then, the following equational condition must be satisfied:

$$\int_{s_1}^{s_2} \boldsymbol{\zeta} \, ds - \boldsymbol{l} = \boldsymbol{0}, \quad \forall \, t \in [t_1, \, t_2]. \tag{6}$$

The orientation at some points of the object may also be controlled during the operation. These orientational constraints are simply described as follows:

$$\boldsymbol{\theta}(s_1, t) - \boldsymbol{\theta}_a = \mathbf{o}, \quad \forall \ t \in [t_1, t_2]$$
(7)

where $\boldsymbol{\theta}_a = [\phi_a \ \theta_a \ \psi_a]^{\mathrm{T}}$ are predefined angles at one operational point $\mathrm{P}(s_1)$.

Equational constraints given by eqs.(6) and (7) can be taken into the optimization problem shown in eq.(5) as follows:

$$F = \int_{t_1}^{t_2} (T - U) \, dt + \sum_i \int_{t_1}^{t_2} \lambda_i \cdot g_i \, dt \qquad (8)$$

where λ_i and g_i represent Lagrange undetermined multiplier functions and the equational conditions, respectively. From the above discussion, the shape of rodlike objects is determined by minimizing the difference between the kinetic energy described in eq.(4) and potential energy described in eq.(3) under geometric constraints described in eqs.(6) and (7). Namely, computation of the shape of rodlike objects results in a variational problem under several conditions.

3 Procedure to Compute Shape of Rodlike Object

In the previous section, we found that the computation of the shape of rodlike objects results in a variational problem. We will establish the procedure to compute the shape of rodlike objects by applying Euler's approach, which is one method to solve a variational problem.

Let $\hat{\alpha}$ be a vector consisting of variables as follows:

$$\boldsymbol{\alpha} = [\phi, \theta, \psi, x, y, z, \lambda_1, \lambda_2, \cdots]$$

$$\stackrel{\Delta}{=} [\alpha_1, \alpha_2, \alpha_3, \cdots].$$

Let vector p be the partial derivative of α with respect to distance s and vector q be the partial derivative of α with respect to time t, namely,

$$p = \frac{\partial \alpha}{\partial s}, \ q = \frac{\partial \alpha}{\partial t}$$

Then, it can be proved that the vector α that minimizes eq.(8) must satisfy the following differential equations:

$$f_k(s,t) = \frac{\partial F}{\partial \alpha_k} - \frac{\partial}{\partial s} \left(\frac{\partial F}{\partial p_k}\right) - \frac{\partial}{\partial t} \left(\frac{\partial F}{\partial q_k}\right) = 0,$$

$$\forall s \in [0, L], \ \forall t \in [t_1, t_2]$$
(9)

where α_k , p_k , and q_k are components of $\boldsymbol{\alpha}$, \boldsymbol{p} , and \boldsymbol{q} , respectively.

If the position and the velocity at each point $P(s,t_1)$ of the object is predetermined, eq.(9) can be regarded as a set of functions of angular acceleration $\partial^2 \phi / \partial t^2$, $\partial^2 \theta / \partial t^2$, and $\partial^2 \psi / \partial t^2$ as follows:

$$f_k\left(\frac{\partial^2 \phi}{\partial t^2}, \frac{\partial^2 \theta}{\partial t^2}, \frac{\partial^2 \psi}{\partial t^2}, s\right)\Big|_{t=t_1} = 0, \quad \forall s \in [0, L].$$

$$\tag{10}$$

It is found that eq.(10) is equivalent to the following equation:

$$\sum_{k} \int_{0}^{L} \left(f_{k} |_{t=t_{1}} \right)^{2} ds = 0.$$
 (11)

Let us express Eulerian angles, ϕ , θ , ψ , and their first and second partial derivatives with respect to time t by linear combinations of basic functions $e_1(s)$ through $e_n(s)$ as follows:

$$\boldsymbol{\theta} = [\boldsymbol{a}_1, \ \boldsymbol{a}_2, \ \boldsymbol{a}_3]^{\mathrm{T}} \boldsymbol{e}(s),$$

$$\frac{\partial \boldsymbol{\theta}}{\partial t} = [\boldsymbol{b}_1, \ \boldsymbol{b}_2, \ \boldsymbol{b}_3]^{\mathrm{T}} \boldsymbol{e}(s),$$

$$\frac{\partial^2 \boldsymbol{\theta}}{\partial t^2} = [\boldsymbol{c}_1, \ \boldsymbol{c}_2, \ \boldsymbol{c}_3]^{\mathrm{T}} \boldsymbol{e}(s)$$

where $\boldsymbol{\theta}$ represents Eulerian angles vector $[\phi \ \theta \ \psi]^{\mathrm{T}}$, $\boldsymbol{a}_i, \boldsymbol{b}_i$, and $\boldsymbol{c}_i \ (i = 1, 2, 3)$ represent coefficient vectors with respect to Eulerian angles, their angular velocity, and their angular acceleration, respectively. And $\boldsymbol{e}(s)$ represents a vector consisting of basic functions $[e_1(s) \ e_2(s) \ \cdots \ e_n(s)]^{\mathrm{T}}$. Then, first and second partial derivatives of Eulerian angles with respect to distance s are represented as follows:

$$\frac{\partial \boldsymbol{\theta}}{\partial s} = [\boldsymbol{a}_1, \ \boldsymbol{a}_2, \ \boldsymbol{a}_3]^{\mathrm{T}} \frac{d}{ds} \boldsymbol{e}(s),$$

$$\frac{\partial^2 \boldsymbol{\theta}}{\partial s^2} = [\boldsymbol{a}_1, \ \boldsymbol{a}_2, \ \boldsymbol{a}_3]^{\mathrm{T}} \frac{d^2}{ds^2} \boldsymbol{e}(s).$$

From the above discussion, eq.(11) can be represented as follows:

$$\sum_{k} \int_{0}^{L} \left\{ f_{k}(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, s) \right|_{t=t_{1}} \right\}^{2} ds = 0, \quad t = t_{1} \quad (12)$$

where $a=[a_1, a_2, a_3]$, $b=[b_1, b_2, b_3]$, and $c=[c_1, c_2, c_3]$. Let us assume that the vector c which minimizes the left side of eq.(12) satisfies eq.(12) because the left side of eq.(12) is a quadratic form of vector c and is not negative. The vector c which minimizes the left side of eq.(12) must satisfy with the following set of equations:

$$\frac{\partial}{\partial c_i} \sum_k \int_0^L \left(f_k |_{t=t_1} \right)^2 ds = 0, \quad i = 1, 2, \cdots, 3n$$
 (13)

Eq.(13) can be represented the linear combination of angular acceleration coefficients c as follows:

$$Ac = d \tag{14}$$

where (3n, 3n) matrix A and (3n) vector d are functions of determined variables a, b, e(s), and their derivatives with respect to s. By solving a set of equations represented by eq.(14), we can derive three angular accelerations $\partial^2 \phi / \partial t^2$, $\partial^2 \theta / \partial t^2$, and $\partial^2 \psi / \partial t^2$, which are described as functions of distance s at $t = t_1$.

Furthermore, the velocity and the position of the object at $t = t_1 + \Delta t$ can be represented approximately as follows:

$$\frac{\partial \boldsymbol{\theta}}{\partial t}\Big|_{t=t_1+\Delta t} \cong \left.\frac{\partial \boldsymbol{\theta}}{\partial t}\right|_{t=t_1} + \Delta t \cdot \left.\frac{\partial^2 \boldsymbol{\theta}}{\partial t^2}\right|_{t=t_1} \tag{15}$$

$$\boldsymbol{\theta}(s, t_1 + \Delta t) \cong \boldsymbol{\theta}(s, t_1),$$

$$+ \Delta t \cdot \frac{\partial \boldsymbol{\theta}}{\partial t} \Big|_{t=t_1} + \frac{(\Delta t)^2}{2} \left. \frac{\partial^2 \boldsymbol{\theta}}{\partial t^2} \right|_{t=t_1}.$$

$$(16)$$

As a result, the shape of rodlike objects can be derived by solving eq.(14) and computing eqs.(15) and (16)repeatedly.

4 Numerical Examples

In this section, we will show some numerical examples by using a proposed procedure in the previous section. The following set of basic functions, which were used in the computation of the static shape of a rodlike object[3], are used in the computation of these examples:

$$e_1(s) = 1, \quad e_2(s) = \frac{s}{L},$$

 $e_{2n+1} = \sin \frac{n\pi s}{L},$
 $e_{2n+2} = \cos \frac{n\pi s}{L}. \quad (n = 1, 2, 3, 4)$

The number of basic functions should be determined with considering both precision in approximation and computing time.

2-Dimensional Deformation without External Force The first example shows dynamic deformation of a rodlike object in 2-dimensional space without an external force. The shape of a rodlike object and its energy then can be described by two of Eulerian angles, that is, $\theta(s, t)$ and $\theta_0(t)$.



Figure 4: Example of 2-dimensional deformation without external force

As shown in Figure 4, let us assume that a rodlike object is deformed so that the distance between two endpoints is equal to 0.8L, that any moment cannot be exerted at both endpoints, and that this object has neither translational nor rotational velocity in initial state. Namely:

$$\begin{aligned} \boldsymbol{x}(L,0) &= \begin{bmatrix} z\\ x \end{bmatrix} = \begin{bmatrix} 0.8L\\ 0 \end{bmatrix}, \\ \frac{\partial\theta(0,t)}{\partial s}\Big|_{t=0} &= \frac{\partial\theta(L,t)}{\partial s}\Big|_{t=0} = 0, \end{aligned}$$
(17)
$$\frac{\partial \boldsymbol{x}}{\partial t}\Big|_{t=0} &= \frac{\partial\theta}{\partial s}\Big|_{t=0} = 0, \quad \forall \ s \in [0, \ L]. \end{aligned}$$

The initial shape of a rodlike object can be derived by minimizing the potential energy consisting of flexural energy alone under constraints given by eq.(17). In detail, see [3]. Furthermore, we assume that the left endpoint of the object fixed on the global coordinate system O-xyz:

$$\boldsymbol{x}_0 = \boldsymbol{0}, \quad \forall \ t \in [0, \ \infty]. \tag{18}$$

Let us release the right endpoint of the object from the geometric constraints given by eq.(17) at t = 0. The equation of motion is then described as follows:

$$-I_{1}\frac{\partial^{2}(\theta-\theta_{0})}{\partial t^{2}} + R_{f}\frac{\partial^{2}\theta}{\partial s^{2}} + \rho\sin\theta\int_{s}^{L}\left\{\frac{\partial^{2}z}{\partial t^{2}} - \left(\frac{\partial\theta_{0}}{\partial t}\right)^{2}(z-z_{c})\right\} ds - \rho\cos\theta\int_{s}^{L}\left\{\frac{\partial^{2}x}{\partial t^{2}} - \left(\frac{\partial\theta_{0}}{\partial t}\right)^{2}(x-x_{c})\right\} ds = 0.$$
(10)

where z_c and x_c represent spatial coordinates of the center of mass. Let c^t be an undetermined coefficients vector at time t as follows:

$$\frac{\partial^2 \theta}{\partial t^2} \bigg|_{t=t} = \sum_{i=1}^{10} c_i^t \cdot e_i(s), \quad \frac{\partial^2 \theta_0}{\partial t^2} \bigg|_{t=t} = c_{11}^t$$

Figure 5 illustrates the computational results corresponding to L = 1.00, $R_f = 1.00 \times 10^2$, $\rho = 1.00 \times 10$, $I_1 = 1.00$, $\Delta t = 1.00 \times 10^{-4}$.



Figure 5: Computational result of 2-dimensional deformation without external force

2-Dimensional Deformation with External Force The second example shows 2-dimensional deformation a rodlike object when an external force is imposed on it.



Figure 6: Example of 2-dimensional deformation with external force

The initial state when the object does not move is same as that of the first example. Furthermore, we assume that its right endpoint can move along z-axis alone:

$$x(L,t) = 0, \quad \forall t \in [0, \infty].$$
(20)

Let us impose an external force f_z constantly on this object at right endpoint in the direction of the z-axis. It is found that the reaction force f_x also exerts at same point in the direction of the x-axis because the geometric constraint given by eq.(20) is imposed on the object at that point. The equation of motion is then described as follows:

$$-I_{1}\frac{\partial^{2}(\theta-\theta_{0})}{\partial t^{2}} + R_{f}\frac{\partial^{2}\theta}{\partial s^{2}} + f_{z}\sin\theta - f_{x}\cos\theta$$
$$+\rho\sin\theta\int_{s}^{L}\left\{\frac{\partial^{2}z}{\partial t^{2}} - \left(\frac{\partial\theta_{0}}{\partial t}\right)^{2}(z-z_{c})\right\} ds$$
$$-\rho\cos\theta\int_{s}^{L}\left\{\frac{\partial^{2}x}{\partial t^{2}} - \left(\frac{\partial\theta_{0}}{\partial t}\right)^{2}(x-x_{c})\right\} ds = 0.$$
(21)

Let c^t be an undetermined coefficients vector which represents the angular acceleration and the reaction force at time t as follows:

$$\left. \frac{\partial^2 \theta}{\partial t^2} \right|_{t=t} = \sum_{i=1}^{10} c_i^t \cdot e_i(s), \quad \left. \frac{\partial^2 \theta_0}{\partial t^2} \right|_{t=t} = c_{11}^t, \quad f_x(t) = c_{12}^t$$

Figure 7 illustrates the computational results corresponding to L = 1.00, $R_f = 1.00 \times 10^2$, $\rho = 1.00 \times 10^2$, $I_1 = 1.00$, $f_z = -4.0 \times 10^4$, $\Delta t = 1.00 \times 10^{-4}$.



Figure 7: Computational result of 2-dimensional deformation with external force

The relationship between time t and a reaction force f_x at the right endpoint of the object are shown in Figure 8. As shown here, we can compute both the shape of dynamic deformation and reaction forces by use of the proposed procedure.

5 Concluding Remarks

In this paper, we analyzed dynamic deformation of



Figure 8: Relationship between time and reaction force at right endpoint of object

rodlike objects. Firstly, the geometric representation of a rodlike object which deforms dynamically was established. It is found that the motion and deformation of a rodlike object can be described by Eulerian angles and their derivatives with respect to the distance s from one endpoint and the time t. Secondly, the potential and the kinetic energy of the object, and the geometric constraints imposed on it were formulated. The shape of dynamic deformed object can be derived by minimizing the time integral of the difference between the kinematic energy and the potential energy under the geometric constraints. Thirdly, a procedure to compute the shape of dynamic deformation was developed. Finally, some numerical examples were shown in order to demonstrate the effectiveness of our proposed procedure.

We believe that this dynamic analysis of rodlike objects will be useful for their quick manipulation.

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