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Modeling of Flexible Belt Objects toward Their Manipulation

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***Abstract*— A modeling method of flexible belt objects is proposed. Deformation of a belt object such as film circuit boards or flexible circuit boards must be estimated for automatic manipulation and assembly. In this paper, we assume that deformation of an inextensible belt object can be described by the shape of its central axis in a longitudinal direction called "the spine line" and lines with zero curvature called "rib lines". This model is referred to as a "fishbone model" in this paper. First, we describe deformation of a rectangular belt object using differential geometry. Next, we propose the fishbone model considering characteristics of a developable surface, i.e., a surface without expansion or contraction. Then, we formulate potential energy of the object and constraints imposed on it. Moreover, this model is applied to a bent belt object. Finally, we explain a procedure to compute the deformed shape of straight and bent objects and compare their shape with measured shape to demonstrate the validity of our proposed method.**

I. Introduction

According to downsizing of various electronic devices such as note PCs, mobile phones, digital cameras, and so on, the more film circuit boards or flexible circuit boards illustrated in Fig.1 are used instead of conventional hard circuit boards. It is difficult to assemble such flexible boards by a robot because they can be easily deformed during their manipulation process and they must be deformed appropriately in the final state. For example, the flexible circuit board shown in Fig.1-(a) must deform to the objective shape illustrated in Fig.1-(b) to install into the hinge part of a mobile phone. Therefore, analysis and estimation of deformation of film/flexible circuit boards is required.

Insertion of a wire into a hole in 2D space has been analyzed using a beam model of the wire to derive a strategy to perform the insertion successfully[1][2]. Kosuge et al. have proposed a control algorithm of dual manipulators handling a flexible sheet metal[3]. In these studies, only bending deformation of a flexible object is considered. Lamiraux et al. have proposed a method of path planning for elastic object manipulation with its deformation to avoid contact with obstacles in a static environment[4]. To eliminate vibration of a linear object during manipulation, an FE model has been applied to formulate its dynamics[5]. Dynamic modeling of a flexible object with an arbitrary shape has been proposed to manipulate it without vibration[6]. In differential geometry, curved lines in 2D or 3D space have been studied to describe their shapes mathematically[7]. Moll et al. have proposed a method to compute the stable shape of a linear object under some geometrical constraints quickly based on differential geometry[8]. It can be applied to path planning for flexible wires. We have proposed a modeling method for linear object deformation based on differential geometry and its applications to manipulative operations[9]. In this method, linear object deformation with flexure, torsion, and extension can be described by only four functions. This method can be applied to a sheet object if the shape of the object is regarded as rectangle, namely, the object has belt-like shape.

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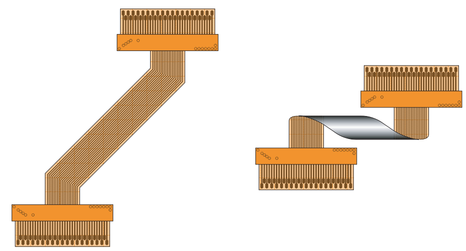


(a) natural shape (b) objective shape

Fig. 1. Example of flexible circuit board

In this paper, we propose *a fishbone model* based on differential geometry to represent belt object deformation. In this model, deformation of a belt object is represented using its central axis in a longitudinal direction referred to as *the spine line* and lines with zero curvature referred to as *rib lines*. The objective of manipulation of a flexible circuit board is to connect its ends to other devices. So, it is important to estimate position and orientation of ends of the board. This implies that we have to estimate more accurately its deformation in a longitudinal direction than that in a transverse direction. The fishbone model is suitable for representation of deformation in a longitudinal direction, that is, deformed shape of the spine line. Moreover, we can estimate belt object deformation if only the flexural rigidity of the object along the spine line is given. It indicates that we can easily identify the actual parameter of the object from experiment. First, we describe deformation of a rectangular belt object using differential geometry. Next, we propose the fishbone model considering characteristics of a developable surface, *i.e.*, a surface without expansion or contraction. After that, we formulate potential energy of the object and constraints imposed on it. Moreover, we apply the fishbone model to a belt object with multiple bends. Finally, we explain a procedure to compute the deformed shape of the object and compare the computed shape of a rectangular belt and an L-shaped belt with their measured shape to demonstrate the validity of our fishbone model.

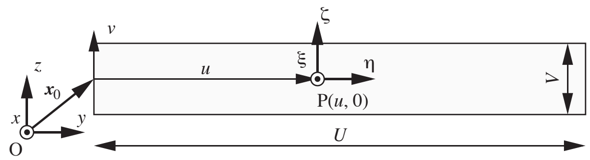


Fig. 2. Coordinates of belt object

II. Modeling of Straight Belt Object

A. Differential Geometry Coordinates

In this section, we formulate the deformation of a rectangular belt object in 3D space. Assumptions in this paper are as follows:

- The width of the belt object is sufficiently small compared to its length.
- The object is inextensible. Namely, it can be bent and twisted but cannot be expanded or contracted.
- Both ends of the object cannot be deformed because connectors are attached to the ends.

In this paper, we focus on deformation of the central axis in a longitudinal direction of a belt object and attempt to represent the whole shape of the object using it.

Let U and V be the length and the width of the object, respectively. Let u be the distance from one end of the object along the central axis in its longitudinal direction and let v be the distance from the central axis in a transverse direction of the object. Let $\mathrm{P}(u, v)$ be a point on the object. In order to describe deformation of the central axis of a belt object, the global space coordinate system and the local object coordinate systems at individual points on the object are introduced as shown in Fig.2. Let O-xyz be the coordinate system fixed in space and P-$\xi\eta\zeta$ be the coordinate system fixed at an arbitrary point $\mathrm{P}(u, 0)$ on the central axis of the object. Assume that the central axis in a longitudinal direction of the object is parallel to the y-axis and the normal vector of any point on the object is parallel to the x-axis in its natural state whereby the object has no deformation. Select the direction of coordinates so that the ξ-, η-, and ζ-axes are parallel to the x-, y-, and z-axes, respectively, in the natural state. Deformation of the object is then represented by the relationship between the local coordinate system P-$\xi\eta\zeta$ at each point on the object and the global coordinate system O-xyz. This is referred to as differential geometry coordinate representation. Let us describe the orientation of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(u, 0)$, $\theta(u, 0)$, and $\psi(u, 0)$. Let $\boldsymbol{\xi}$, $\boldsymbol{\eta}$, and $\boldsymbol{\zeta}$ be unit vectors along the ξ-, η-, and ζ-axes, respectively, at point $\mathrm{P}(u, 0)$. These unit vectors are then described by

$$\left[\ \boldsymbol{\xi} \mid \boldsymbol{\eta} \mid \boldsymbol{\zeta}\ \right] = \left[\begin{array}{c|c|c} C_\theta C_\phi C_\psi - S_\phi S_\psi & -C_\theta C_\phi S_\psi - S_\phi C_\psi & S_\theta C_\phi \\ C_\theta S_\phi C_\psi + C_\phi S_\psi & -C_\theta S_\phi S_\psi + C_\phi C_\psi & S_\theta S_\phi \\ -S_\theta C_\psi & S_\theta S_\psi & C_\theta \end{array}\right]. \quad (1)$$

For the sake of simplicity, $\cos\theta$ and $\sin\theta$ are abbreviated as C_θ and S_θ, respectively, for example. Note that the Eulerian angles depend on distance u. Let $\boldsymbol{x}(u, 0) = [\, x(u, 0),\ y(u, 0),\ z(u, 0)\,]^{\mathrm{T}}$ be the position vector of point $\mathrm{P}(u, 0)$. The position vector can be computed by integrating vector $\boldsymbol{\eta}(u, 0)$. Namely,

$$\boldsymbol{x}(u, 0) = \boldsymbol{x}_0 + \int_0^u \boldsymbol{\eta}(u, 0)\, \mathrm{d}u, \quad (2)$$

where $\boldsymbol{x}_0 = [\, x_0,\ y_0,\ z_0\,]^{\mathrm{T}}$ is the position vector at the end point $\mathrm{P}(0, 0)$.

Let $\omega_{u\xi}$, $\omega_{u\eta}$, and $\omega_{u\zeta}$ be infinitesimal ratios of rotation angles around the ξ-, η-, and ζ-axes, respectively, at point $\mathrm{P}(u, 0)$. Then, the infinitesimal angle ratios can be described as follows:

$$\begin{bmatrix} \omega_{u\xi} \\ \omega_{u\eta} \\ \omega_{u\zeta} \end{bmatrix} = \begin{bmatrix} -S_\theta C_\psi \\ S_\theta S_\psi \\ C_\theta \end{bmatrix} \frac{\mathrm{d}\phi}{\mathrm{d}u} + \begin{bmatrix} S_\psi \\ C_\psi \\ 0 \end{bmatrix} \frac{\mathrm{d}\theta}{\mathrm{d}u} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \frac{\mathrm{d}\psi}{\mathrm{d}u}. \quad (3)$$

B. Description of Surface Bending

Next, we consider general description of 3D surface. Let $\boldsymbol{x}(u, v)$ be the position vector of point $\mathrm{P}(u, v)$ on a surface. Let $\boldsymbol{x}_u(u, v)$ and $\boldsymbol{x}_v(u, v)$ be tangent vectors at point $\mathrm{P}(u, v)$ along u- and v-axes, respectively, and let $\boldsymbol{e}(u, v)$ be the normal vector at point $\mathrm{P}(u, v)$. According to differential geometry, the normal curvature κ in direction $\boldsymbol{d} = a\boldsymbol{x}_u + b\boldsymbol{x}_v$ is represented as follows:

$$\kappa = \frac{La^2 + 2Mab + Nb^2}{Ea^2 + 2Fab + Gb^2}, \quad (4)$$

where E, F, and G are coefficients of the first fundamental form and L, M, and N are those of the second fundamental form of the surface. These coefficients are defined as follows:

$$E = \boldsymbol{x}_u \cdot \boldsymbol{x}_u, \quad F = \boldsymbol{x}_u \cdot \boldsymbol{x}_v, \quad G = \boldsymbol{x}_v \cdot \boldsymbol{x}_v, \quad (5)$$

$$L = \frac{\partial \boldsymbol{x}_u}{\partial u} \cdot \boldsymbol{e}, \quad M = \frac{\partial \boldsymbol{x}_u}{\partial v} \cdot \boldsymbol{e}, \quad N = \frac{\partial \boldsymbol{x}_v}{\partial v} \cdot \boldsymbol{e}. \quad (6)$$

The normal curvature κ depends on the direction $\boldsymbol{d}$ and its maximum value κ_1 and its minimum value κ_2 are called the principal curvatures. Direction $\boldsymbol{d}_1$ of the maximum curvature κ_1 and direction $\boldsymbol{d}_2$ of the minimum curvature κ_2 are referred to as principal directions. The principal curvatures and the principal directions specify bending of a surface. A surface is also characterized by Gaussian curvature $K(u, v)$ and the mean curvature $H(u, v)$. They are related to the principal curvatures κ_1 and κ_2 by

$$K = \kappa_1 \kappa_2 = \frac{LN - M^2}{EG - F^2}, \quad (7)$$

$$H = \frac{\kappa_1 + \kappa_2}{2} = \frac{EN - 2FM + GL}{2(EG - F^2)}. \quad (8)$$

Vectors $\boldsymbol{x}_u$, $\boldsymbol{x}_v$, and $\boldsymbol{e}$ correspond to $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$, and $\boldsymbol{\xi}$ in this paper, respectively. Then, coefficients of the first fundamental form are $E = 1$, $F = 0$, and $G = 1$, respectively. Moreover, the derivation of unit vectors $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ can be described using infinitesimal ratios of rotation angles as follows:

$$\frac{\partial \boldsymbol{\eta}}{\partial u} = -\omega_{u\zeta}\boldsymbol{\xi} + \omega_{u\xi}\boldsymbol{\zeta}, \quad (9)$$

$$\frac{\partial \boldsymbol{\zeta}}{\partial u} = \omega_{u\eta}\boldsymbol{\xi} - \omega_{u\xi}\boldsymbol{\eta} = \frac{\partial \boldsymbol{\xi}}{\partial v}. \quad (10)$$

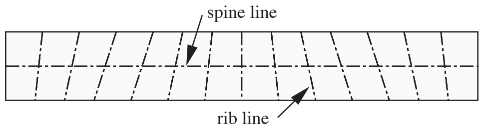


Fig. 3. Fishbone model

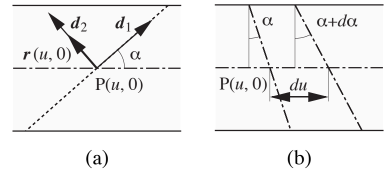


Fig. 4. Rib angle and rib lines

Substituting eqs.(9) and (10) into eqs.(6), L and M can be represented as a function of infinitesimal angle ratios as follows:

$$L = (-\omega_{u\zeta}\boldsymbol{\xi} + \omega_{u\xi}\boldsymbol{\zeta}) \cdot \boldsymbol{\xi} = -\omega_{u\zeta}, \tag{11}$$

$$M = (\omega_{u\eta}\boldsymbol{\xi} - \omega_{u\xi}\boldsymbol{\eta}) \cdot \boldsymbol{\xi} = \omega_{u\eta}. \tag{12}$$

In contrast, N cannot be described by Eulerian angles. So, we introduce the fourth parameter $\delta(u,0)$: $N = \delta(u,0)$. It corresponds to the curvature in a transverse direction. Consequently, Gaussian curvature K and the mean curvature H is described by

$$K = -\omega_{u\zeta}\delta - \omega_{u\eta}^2, \tag{13}$$

$$H = \frac{\delta - \omega_{u\zeta}}{2}. \tag{14}$$

Thus, bending of a surface is characterized by Eulerian angles $\phi(u,0)$, $\theta(u,0)$, and $\psi(u,0)$ and the curvature in a transverse direction $\delta(u,0)$. Note that K and H depends on not only coordinate u but also coordinate v. In this paper, we assume that the whole shape of a belt object can be described by the shape of the central axis in a longitudinal direction because the width of a belt object is sufficiently small compared to its length.

If a principal curvature κ_2, *i.e.*, the minimum value of the normal curvature is equal to zero, the surface is developable. Namely, it can be flattened without its expansion or contraction. Such surface is referred to as a *developable surface*. In this paper, we assume that a belt object is inextensible. Then, the deformed shape of the object corresponds to a developable surface. It means that the object bends in direction $\boldsymbol{d}_1$ and it is not deformed in direction $\boldsymbol{d}_2$. Namely, a line the direction of which coincides with direction $\boldsymbol{d}_2$ is kept straight after deformation. In this paper, the central axis in a longitudinal direction of the object is referred to as the *spine line* and a line with zero curvature at a point on the object is referred to as a *rib line* as shown in Fig.3. We assume that bending and torsion of the spine line and direction of the rib line of each point specifies deformation of a belt object. This model is referred to as a *fishbone model* in this paper. Let $\alpha(u,0)$ be rib angle, which is the angle between the spine line and direction $\boldsymbol{d}_1$ as shown in Fig.4-(a). Consequently, the shape of a straight belt object can be represented using five variables $\phi(u)$, $\theta(u)$, $\psi(u)$, $\delta(u)$, and $\alpha(u)$. Note that they depend on only the distance u from one end of the object along the spine line.

C. Constraints on Belt Object Variables

Let us consider conditions which five variables must satisfy so that the surface of a belt object is developable. Gaussian curvature K of a developable surface must be zero at any point. So, the following constraint is imposed on the object.

$$K = -\omega_{u\zeta}\delta - \omega_{u\eta}^2 = 0, \quad \forall u \in [\ 0,\ U\ ]. \tag{15}$$

From eq.(15), δ is described by

$$\delta = -\frac{\omega_{u\eta}^2}{\omega_{u\zeta}}. \tag{16}$$

The infinitesimal ratio of rotation angle around ξ-axis $\omega_{u\xi}$ must also be satisfied the following equation because of the inextensibility of a belt object:

$$\omega_{u\xi} = 0, \quad \forall u \in [\ 0,\ U\ ]. \tag{17}$$

Moreover, as shown in Fig.4-(b), to prevent rib lines from intersecting with themselves on a belt object, the following inequalities must be satisfied:

$$-\frac{2\cos^2\alpha}{V} \leq \frac{\mathrm{d}\alpha}{\mathrm{d}u} \leq \frac{2\cos^2\alpha}{V}, \ \ \forall u \in [\ 0,\ U\ ]. \tag{18}$$

Substituting eq.(16) into eqs.(4) and (14), the normal curvature in direction $\boldsymbol{d}_1 = \boldsymbol{\xi}\cos\alpha + \boldsymbol{\eta}\sin\alpha$, *i.e.*, a principal curvature κ_1 is as follows:

$$\begin{aligned}\kappa_1 &= -\omega_{u\zeta}\cos^2\alpha + 2\omega_{u\eta}\cos\alpha\sin\alpha - \frac{\omega_{u\eta}^2}{\omega_{u\zeta}}\sin^2\alpha \\ &= -\omega_{u\zeta} - \frac{\omega_{u\eta}^2}{\omega_{u\zeta}}\end{aligned} \tag{19}$$

Then, α can be described as follows:

$$\alpha = -\tan^{-1}\frac{\omega_{u\eta}}{\omega_{u\zeta}}. \tag{20}$$

Now, let us introduce parameter $\beta(u)$:

$$\beta = \tan\alpha. \tag{21}$$

Then, β must satisfy the following equation from eq.(20):

$$\omega_{u\eta} + \omega_{u\zeta}\beta = 0, \ \ \forall u \in [\ 0,\ U\ ]. \tag{22}$$

Moreover, eq.(18) is described as follows by substituting eq.(21):

$$-\frac{2}{V} \leq \frac{\mathrm{d}\beta}{\mathrm{d}u} \leq \frac{2}{V}, \ \ \forall u \in [\ 0,\ U\ ]. \tag{23}$$

Consequently, the shape of a belt object can be represented by four variables $\phi(u)$, $\theta(u)$, $\psi(u)$, and $\beta(u)$ considering necessary constraints for developability of the object surface. And, they must satisfy eqs.(17), (22), and (23) in any state to maintain developability.

D. Potential Energy and Geometric Constraints

Let us formulate the potential energy of a deformed belt object. We can assume that a belt object bends along direction $\boldsymbol{d}_1$ without torsional deformation. Then, the potential energy I can be described as follows assuming that the flexural energy is proportional to the bending moment at each point $\mathrm{P}(u)$:

$$I = \int_0^U \frac{R_f}{2\cos\alpha} \frac{(\omega_{u\zeta}^2 + \omega_{u\eta}^2)^2}{\omega_{u\zeta}^2} \mathrm{d}u = \int_0^U \frac{R_f}{2} \frac{\omega_{u\zeta}^2}{\cos^5\alpha} \mathrm{d}u, \tag{24}$$

where R_f represents the flexural rigidity of a belt object along the spine line at point $\mathrm{P}(u)$.

Next, let us formulate geometric constraints imposed on a belt object. Let $\boldsymbol{l} = [\, l_x,\ l_y,\ l_z\,]^T$ be a predetermined vector describing the relative position between two operational points on the spine line, $\mathrm{P}(u_a)$ and $\mathrm{P}(u_b)$. Recall that the spatial coordinates corresponding to distance u are given by eq.(2). Thus, the following equation must be satisfied:

$$\boldsymbol{x}(u_b) - \boldsymbol{x}(u_a) = \boldsymbol{l}. \tag{25}$$

The orientational constraint at operational point $\mathrm{P}(u_c)$ is simply described as follows:

$$\phi(u_c) = \phi_c,\ \ \theta(u_c) = \theta_c,\ \ \psi(u_c) = \psi_c, \tag{26}$$

where ϕ_c, θ_c, and ψ_c are predefined Eulerian angles at this point.

Therefore, the shape of a belt object is determined by minimizing the potential energy described by eq.(24) under necessary constraints for developability described by eqs.(17), (22), and (23) and geometric constraints imposed on the object described by eqs.(25) and (26). Namely, computation of the deformed shape of the object results in a variational problem under equational and inequality constraints.

III. Modeling of Belt Object with Bends

Some flexible circuit boards bend like a polygonal line as shown in Fig.1. In this section, we apply the fishbone model to an L- or V-shaped belt object. Fig.5 shows a belt object with one bend. Let λ be the bending angle between the spine line of the left part and that of the right part at point $\mathrm{P}(u_h)$. Line ab illustrated in Fig.5 is referred to as *a connecting line* in this paper. To represent the bend of the object, Eulerian angles and rib angles of the left part and those of the right part are defined separately, for example,

$$\phi(u) = \begin{cases} \phi_1(u) & (0 \le u \le u_h)\,, \\ \phi_2(u) & (u_h \le u \le U)\,, \end{cases} \tag{27}$$

where a parameter with subscript 1 is for the left part and that with subscript 2 for the right part of the object, respectively. Then, the following conditions must be satisfied at point $\mathrm{P}(u_h)$:

$$\boldsymbol{\eta}_2\,(u_h) = \boldsymbol{\eta}_1\,(u_h)\cos\lambda + \boldsymbol{\zeta}_1\,(u_h)\sin\lambda, \tag{28}$$
$$\boldsymbol{\zeta}_2\,(u_h) = -\boldsymbol{\eta}_1\,(u_h)\sin\lambda + \boldsymbol{\zeta}_1\,(u_h)\cos\lambda. \tag{29}$$

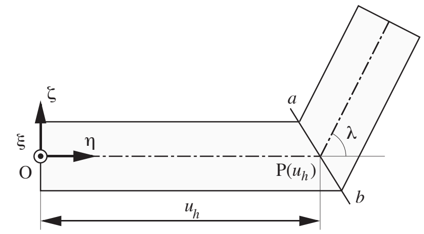


Fig. 5. Object with one bend

Let us assume that the rib line at point $\mathrm{P}(u_h)$ coincides with the connecting line, namely,

$$\beta_1\,(u_h) = \tan(\lambda/2), \tag{30}$$
$$\beta_2\,(u_h) = -\tan(\lambda/2). \tag{31}$$

Moreover, the principal curvature κ_1 at point $\mathrm{P}(u_h)$ must be continued. So, the following condition is derived:

$$\omega_{u\zeta1}\,(u_h) + \frac{\omega_{u\eta1}^2\,(u_h)}{\omega_{u\zeta1}\,(u_h)} = \omega_{u\zeta2}\,(u_h) + \frac{\omega_{u\eta2}^2\,(u_h)}{\omega_{u\zeta2}\,(u_h)}. \tag{32}$$

The potential energy I of this bent object is described by

$$I = \int_0^{u_h} \frac{R_f}{2} \frac{\omega_{u\zeta1}^2}{\cos^5\alpha_1} \mathrm{d}u + \int_{u_h}^U \frac{R_f}{2} \frac{\omega_{u\zeta2}^2}{\cos^5\alpha_2} \mathrm{d}u. \tag{33}$$

Then, the deformed shape of the object is derived by minimizing the above potential energy under not only some geometric constraints but also constraints described by eqs.(28) through (32). This model also can be applied to a belt object with multiple bends.

Flexible circuit boards with curves also exist. Note that the infinitesimal ratio of rotation angle around ξ-axis $\omega_{u\xi}$ corresponds to the curvature of the spine line of a belt object. As we assume that the spine line is straight in this paper, it is constantly equal to zero. If an object is curved with a certain curvature, $\omega_{u\xi}$ must be equal to that curvature even if the object deforms. We can impose this constraint on the object instead of eq.(17). This implies that our proposed method can be applied to a curved belt object.

IV. Computation of Belt Object Deformation

A. Computation Algorithm

Computation of the deformed shape of a belt object results in a variational problem as mentioned in Section II and III. In [9], we developed an algorithm based on Ritz's method[10] and a nonlinear programming technique to compute linear object deformation. In this paper, we apply such algorithm to the computation of belt object deformation.

Let us express functions $\phi(u)$, $\theta(u)$, $\psi(u)$, and $\beta(u)$ by linear combinations of basis functions $e_1(u)$ through $e_n(u)$:

$$\phi(u) = \sum_{i=1}^{n} a_i^{\phi} e_i(u) \triangleq \boldsymbol{a}^{\phi} \cdot \boldsymbol{e}(u), \tag{34}$$

$$\theta(u) = \sum_{i=1}^{n} a_i^{\theta} e_i(u) \triangleq \boldsymbol{a}^{\theta} \cdot \boldsymbol{e}(u), \tag{35}$$

$$\psi(u) = \sum_{i=1}^{n} a_i^{\psi} e_i(u) \triangleq \boldsymbol{a}^{\psi} \cdot \boldsymbol{e}(u), \tag{36}$$

$$\beta(u) = \sum_{i=1}^{n} a_i^{\beta} e_i(u) \stackrel{\triangle}{=} \boldsymbol{a}^{\beta} \cdot \boldsymbol{e}(u), \tag{37}$$

where $\boldsymbol{a}^{\phi}$, $\boldsymbol{a}^{\theta}$, $\boldsymbol{a}^{\psi}$, and $\boldsymbol{a}^{\beta}$ are vectors consisting of coefficients corresponding to functions $\phi(u)$, $\theta(u)$, $\psi(u)$, and $\beta(u)$ respectively, and vector $\boldsymbol{e}(u)$ is composed of basis functions $e_1(u)$ through $e_n(u)$. Substituting the above equations into eq.(24), potential energy I is described by a function of coefficient vectors $\boldsymbol{a}^{\phi}$, $\boldsymbol{a}^{\theta}$, $\boldsymbol{a}^{\psi}$, and $\boldsymbol{a}^{\beta}$. Constraints are also described by conditions involving the coefficient vectors. Especially, discretizing eqs.(17), (22), and (23) by dividing interval $[0, U]$ into n small intervals yields a finite number of conditions. As a result, a set of the constraints is expressed by equations and inequalities in terms of the coefficient vectors.

Consequently, the deformed shape of a belt object can be derived by computing a set of coefficient vectors $\boldsymbol{a}^{\phi}$, $\boldsymbol{a}^{\theta}$, $\boldsymbol{a}^{\psi}$, and $\boldsymbol{a}^{\beta}$ that minimizes the potential energy under the constraints. This minimization problem can be solved by the use of a nonlinear programming technique such as the multiplier method[11]. In case of a belt object with one bend, coefficient vectors for the left part and those for the right part are defined separately, for example,

$$\phi_1(u) \stackrel{\triangle}{=} \boldsymbol{a}_1^{\phi} \cdot \boldsymbol{e}(u), \;\; \phi_2(u) \stackrel{\triangle}{=} \boldsymbol{a}_2^{\phi} \cdot \boldsymbol{e}(u). \tag{38}$$

Then, coefficient vectors $\boldsymbol{a}_j^{\phi}$, $\boldsymbol{a}_j^{\theta}$, $\boldsymbol{a}_j^{\psi}$, and $\boldsymbol{a}_j^{\beta}$ $(j = 1, 2)$ minimizing the potential energy determine the object shape.

B. Examples of Computation

In this section, numerical examples demonstrate how the proposed method computes the deformed shape of a belt object. The following set of basis functions are used in the computation of these examples:

$$\begin{aligned} &e_1 = 1, \;\; e_2 = u, \\ &e_{2i+1} = \sin\frac{\pi i u}{U}, \;\; e_{2i+2} = \cos\frac{\pi i u}{U}, \;\; (i = 1, 2, 3, 4). \end{aligned} \tag{39}$$

Necessary constraints for developability described by eqs.(17), (22), and (23) are divided into 16 conditions at point P$(iU/15)$ $(i = 0, \cdots, 15)$ respectively in the following examples. All computations were performed on a 750MHz Alpha 21264 CPU with 512MB memory operated by Tru64UNIX. Programs were compiled by a Compaq C Compiler V6.1 with optimization option -O4.

Fig.6 shows the first example of straight object deformation. The length of the object U is equal to 1, its width V is equal to 0.1, and its flexural rigidity along the spine line R_f is constantly equal to 1. In this example, both ends of the spine line are on the same line but directions of the spine line at these points are different. Fig.7 shows computational results. Fig.7-(a), -(b), and -(c) illustrate the top, front, and side view of the object, respectively. As shown in this figure, the object satisfies the given geometric constraints by twisting partially. This computation took about 1,500 seconds.

Fig.8 shows the second example of L-shaped object deformation. The original shape of the object is illustrated in Fig.8-(a). It is 2 long, 0.2 wide, and it has one rectangular bend on its mid point, namely, $u_h = 0.5U$ and $\lambda = \pi/2$. Its flexural rigidity R_f is constantly equal to 1. Positional and orientational constraints are shown in Fig.8-(b). Fig.9 shows computational results. As shown in this figure, the object is not also bent but also twisted and its shape becomes asymmetrical. This computation took about 1,500 seconds.

Thus, our method can estimate bending and torsional deformation of a rectangular and L-shaped belt object using only flexural rigidity of the object along its spine line if the object is isotropic. This flexural rigidity can be measured by a simple experiment.

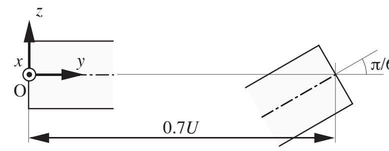
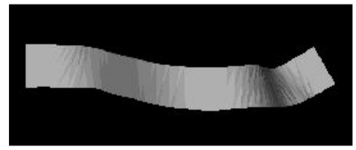
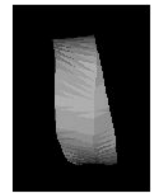


Fig. 6. Example 1



(a) Top view

(b) Front view

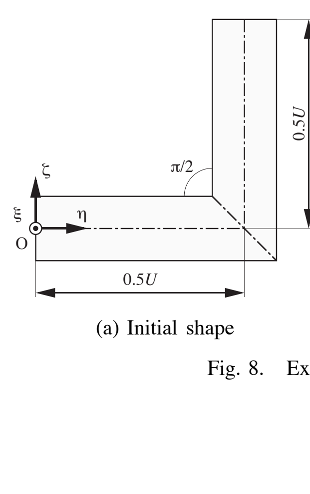


(c) Side view

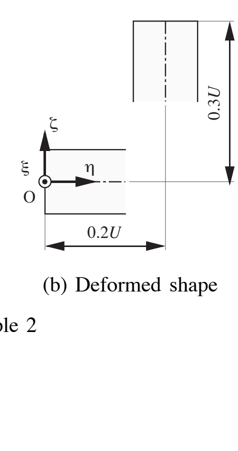
Fig. 7. Computational result of example 1

V. EXPERIMENTS

In this section, the computation results will be experimentally verified by measuring the deformed shape of a belt object. We measured the shape of two types of flexible polystyrol sheets with a 3D scanner. One is a rectangle 200mm long and 20mm wide. The other is L-shaped and each wing is 200mm long and 20mm wide. Both are 140μm thick. Their flexural rigidity is unknown but from eq.(24), it is found that the deformed shape is independent of it

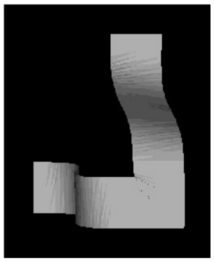


(a) Initial shape

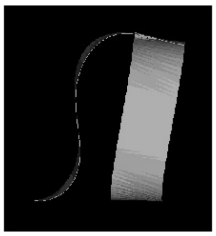


(b) Deformed shape

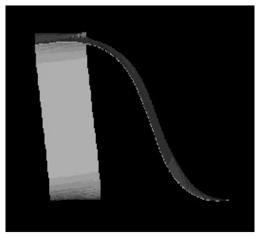
Fig. 8. Example 2



(a) Top view

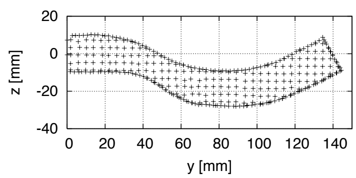


(b) Front view

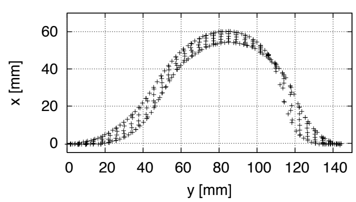


(c) Side view

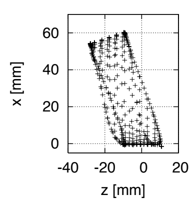
Fig. 9. Computational result of example 2



(a) Top view



(b) Front view



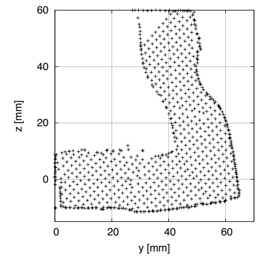
(c) Side view

Fig. 10. Experimental result of example 1

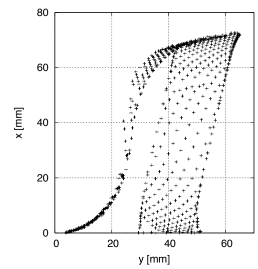
when it is constant along the spine line. Fig.10 shows the experimental result of rectangular belt object deformation. As shown in this figure, the computational result illustrated in Fig.7 is similar to the actual shape. Next, Fig.11 shows the measured shape of the L-shaped belt object. Comparing this shape with the computed shape shown in Fig.9, orientation of the bend point is different. This difference may be caused by the assumption that a rib line coincides with the connecting line at the bend point. So, the validity of this assumption should be more discussed.

VI. Conclusions

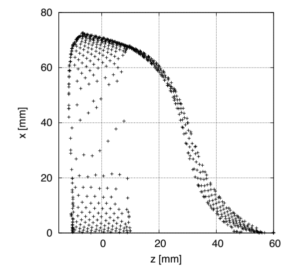
A fishbone model based on differential geometry to represent belt object deformation was proposed toward manipulation/assembly of film/flexible circuit boards. First, deformation of a rectangular belt object was described using differential geometry. Next, the fishbone model was proposed by considering characteristics of a developable surface. In this model, deformation of a belt object is represented using the shape of the spine line and the direction of straight



(a) Top view



(b) Front view



(c) Side view

Fig. 11. Experimental result of example 2

rib lines. Then, we can estimate belt object deformation if only the flexural rigidity of the object along the spine line is given. After that, we formulated potential energy of the object and constraints imposed on it. Moreover, we applied the fishbone model to a belt object with bends. Finally, a procedure to compute the deformed shape of the object was explained and some computational results were compared with experimental results. They demonstrated that the fishbone model can represent deformation of a belt object qualitatively well. Our future works is speeding up of the computation.

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