Dynamic Analysis of Linear Object Deformation and Its Application to Manipulation Planning

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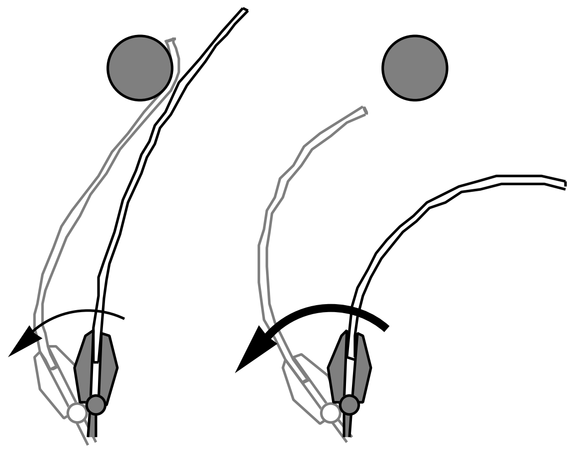
ABSTRACT

A dynamic motion analysis of deformable linear objects is presented. In manufacturing, evaluation of the shape of deformable objects is important for planning of their manipulation. Especially, if deformable objects are operated quickly, the dynamical effect of them cannot be neglected when we evaluate their shapes. In this paper, we analyse dynamic deformation of deformable linear objects. Firstly, a geometric representation of the shape of a linear object is introduced and its potential energy and kinetic energy are formulated. Then, its equations of motion can be derived based on Hamilton's principle. Secondly, a procedure to compute the shape is developed. By using it, an numerical example is presented. Thirdly, the validity of our method is demonstrated with a measuring experiment. Finally, it is shown that our method can be applied to manipulation planning utilizing dynamic deformation of a linear object.

1. INTRODUCTION

In manufacturing, automation of handling and manipulative processes which deal with deformable objects such as wires, cords, rubber tubes, and so on has been done but it is not enough to satisfy our requests. In manipulative processes, if the shape of deformed objects can be predicted, we can operate them successfully by utilizing their deformation. Modeling of deformable objects is thus necessary in order to evaluate their shape on a computer in advance. Especially, modeling of dynamic deformation becomes important because the dynamical effect of them cannot be neglected if they are operated quickly by humans or machines. Furthermore, by considering dynamic deformation, we can derive new task strategies which cannot be obtained when only static deformation is considered. For example, when we manipulate a linear object slowly as shown in Figure 1-(a), it will collide against an obstacle. But, quick manipulation can avoid collision as shown in Figure 1-(b) even if a manipulator tracks the same trajectory. Therefore, it is important for quick manipulation of deformable objects to evaluate the shape of them which deform dynamically in advance.



(a) Slow manipulation (b) Quick manipulation

Figure 1 Example of manipulation utilizing dynamic deformation of linear object

In this paper, we will analyse dynamic deformation of deformable linear objects. First, the geometric shape of a linear object is formulated by using Eulerian angles. Its potential energy and kinetic energy are also formulated. Then, equations of motion can be derived from the formulation by applying Hamilton's principle and Euler's approach. Secondly, a procedure to compute its shape is proposed and a computational result is shown. Thirdly, the validity of our proposed method is demonstrated by comparing the computed shape with the actual shape measured by a high-speed camera. Finally, we try applying our method to manipulation planning utilizing dynamic deformation of a linear object.

2. MODELING OF LINEAR OBJECT DEFORMATION

2.1 Geometric Representation of Deformed Linear Objects

In this section, we will formulate the geometrical shape of a linear object, which moves and deforms dynamically in 3-dimensional space. Let L be the length of the object, s be the distance from one endpoint of the object along it, and t be the time. Let us introduce the global space coordinate system and the local object coordinate systems at individual points on the object and at each time, as shown in Figure 2, in order to describe motion and deformation of a linear object. Let O-xyz be the coordinate system fixed on space and P(s, t)-$\xi\eta\zeta$ be the coordinate system fixed on an arbitrary point of the object at distance s and time t. Select the direction of the local coordinate system P(s, t)-$\xi\eta\zeta$ so that ζ-axis is aligned with the central axis of the object. Then, motion and deformation of the object are represented by the relationship between the local coordinate system P(s,t)-$\xi\eta\zeta$ and the global coordinate system O-xyz. Let us describe the orientation of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(s,t)$, $\theta(s,t)$, and $\psi(s,t)$.

Let $\boldsymbol{x} = [\ x(s,t) \quad y(s,t) \quad z(s,t)\ ]^T$ be spatial coordinates corresponding to point P(s,t) . Then, it can be described by using Eulerian angles $Q = [\ \phi(s,t) \quad \theta(s,t) \quad \psi(s,t)\ ]^T$ as follows:

$$\boldsymbol{x} = \boldsymbol{x}_0 + \int_0^s \zeta \ ds = \boldsymbol{x}_0 + \int_0^s \begin{bmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{bmatrix} ds \tag{1}$$

where $\boldsymbol{x}_0$ denotes the coordinate at the end point corresponding to s=0, which is represented as a function of time t and ζ is the unit vector along ζ-axis.

Let us describe the curvature of the object and its torsional ratio at time t in order to express bending and torsional deformation of the object. Let $\kappa(s,t)$ and $\chi(s,t)$ be the curvature and the torsional ratio at point P(s,t), respectively. They can be described by use of Eulerian angles ϕ, θ, and ψ as follows:

$$\kappa^2 = \left(\frac{\partial\theta}{\partial s}\right)^2 + \left(\frac{\partial\phi}{\partial s}\right)^2 \sin^2\theta, \quad \chi^2 = \left\{\frac{\partial\psi}{\partial s} + \frac{\partial\phi}{\partial s}\cos\theta\right\}^2 \tag{2}$$

Next, let us describe the velocity and the angular velocity of the object at time t, in order to express motion of the object. Let $\boldsymbol{v}$ be velocities of the object at the point P(s,t), namely, $\boldsymbol{v} = \partial\boldsymbol{x}/\partial t$. Furthermore, let $\omega_1(s,t)$ and $\omega_2(s,t)$ be the angular velocity for deformation around the axis which intersects with the central axis perpendicularly, and that around the central axis at point P(s,t), respectively as shown in Figure 3. It is found that these two angular velocities can be described by use of Eulerian angles ϕ, θ, and ψ as follows:

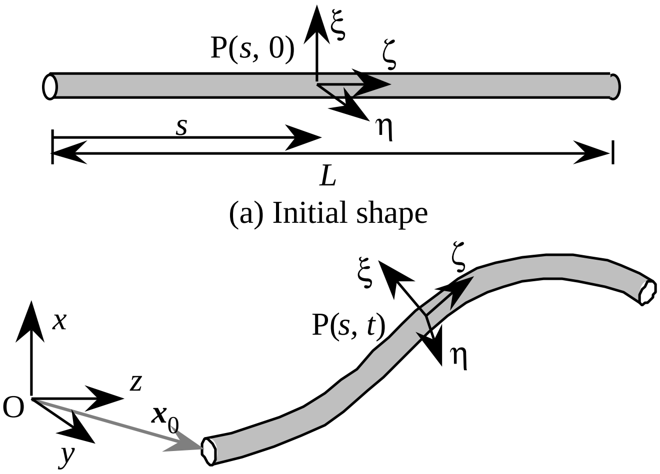
$$\omega_1^2 = \left(\frac{\partial\theta}{\partial t}\right)^2 + \left(\frac{\partial\phi}{\partial t}\right)^2 \sin^2\theta, \quad \omega_2^2 = \left\{\frac{\partial\psi}{\partial t} + \frac{\partial\phi}{\partial t}\cos\theta\right\}^2 . \tag{3}$$

From the above discussion, the geometric shape of a moving and deforming linear object can be represented by three variables, ϕ, θ, and ψ, which depend upon distance s and time t.

2.2 Potential Energy, Kinetic Energy, and Geometric Constraints

In this paper, we will adopt Hamilton's principle that the time integral of the difference between kinetic energy of a object and its potential energy reaches to the minimum when the object moves dynamically. In this section, we will formulate potential energy and kinetic energy of a linear object and geometric constraints imposed on it.

First, let us formulate potential energy of a linear object by considering its bending and torsional deformation. Applying Bernoulli and Navier's assumption, we can describe the potential energy U at time t as follows:



(b) Deformed shape

Figure 2 Motion and deformation of linear object

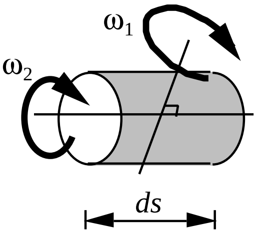


Figure 3 Angular velocities of linear object

$$U(t) = \int_0^L \frac{R_f}{2} \kappa^2 ds + \int_0^L \frac{R_t}{2} \chi^2 ds \tag{4}$$

where R_f and R_t represent flexural rigidity and torsional rigidity at point P(s,t), respectively.

Next, let us formulate kinetic energy of a linear object. Considering translation and rotation of the object, kinetic energy T at time t is described as follows:

$$T(t) = \int_0^L \frac{\rho}{2} \boldsymbol{v}^2 ds + \int_0^L \frac{I_1}{2} \omega_1^2 \, ds + \int_0^L \frac{I_2}{2} \omega_2^2 \, ds \tag{5}$$

where ρ, I_1, and I_2 represent linear density, moment of inertia around the axis which intersects with the central axis of the object perpendicularly, and that around the central axis of it, respectively.

By applying Hamilton's principle, the following functional should reach to its minimum at actual motion and deformation:

$$F = \int_{t_1}^{t_2} (T - U) \, dt \, . \tag{6}$$

Some geometric constraints are imposed on a linear object because it may interact with other objects such as fingertips and obstacles. Such constraints can be represented as equational conditions with respect to ϕ, θ, and ψ. Then, they can be taken into the optimization problem shown in eq.(6) as follows:

$$F = \int_{t_1}^{t_2} (T - U) \, dt + \sum_i \int_{t_1}^{t_2} \lambda_i \cdot g_i \; dt \tag{7}$$

where λ_i and g_i represent Lagrange undetermined multiplier functions and equational conditions, respectively.

From the above discussion, the shape of a linear object is determined by minimizing the difference between kinetic energy described in eq.(5) and potential energy described in eq.(4) under geometric constraints. Namely, computation of the shape of a linear object results in a variational problem under several conditions.

3. PROCEDURE TO COMPUTE LINEAR OBJECT SHAPE

In the previous section, we found that the computation of the shape of a linear object results in a variational problem. We will establish a procedure to compute the shape of a linear object. Differential equations can be derived by applying Euler's approach, which is one method to solve a variational problem. The following equations of motion are derived by applying Euler's approach to eq.(7):

$$f\left(Q, \frac{\partial Q}{\partial s}, \frac{\partial^2 Q}{\partial s^2}, \frac{\partial Q}{\partial t}, \frac{\partial^2 Q}{\partial t^2} \right) = 0, \text{ subject to } \forall s \in [0, L], \ \forall t \in [t_1, t_2]. \tag{8}$$

Eq.(8) is equivalent to the following equation:

$$F = \int_0^L f^2 ds = 0, \text{ subject to } \forall t \in [t_1, t_2]. \tag{9}$$

Let us express Eulerian angles and their first and second partial derivatives by linear combinations of basic functions $e_1(s)$ through $e_n(s)$ as follows:

$$Q = A\boldsymbol{e}(s), \ \frac{\partial Q}{\partial s} = A\frac{d}{ds}\boldsymbol{e}(s), \ \frac{\partial^2 Q}{\partial s^2} = A\frac{d^2}{ds^2}\boldsymbol{e}(s), \ \frac{\partial Q}{\partial t} = B\boldsymbol{e}(s), \ \frac{\partial^2 Q}{\partial t^2} = C\boldsymbol{e}(s) \tag{10}$$

where A, B, and C are $3 \times n$ coefficient matrix, and $\boldsymbol{e}(s)$ represents a vector consisting of basic functions $[\ e_1(s) \quad e_2(s) \quad \ldots \quad e_n(s)\ ]^T$. Then, eq.(9) can be represented as follows:

$$F(A, B, C) = 0, \text{ subject to } \forall t \in [t_1, t_2]. \tag{11}$$

Therefore, if matrices A and B which correspond angles and angular velocities at time t_1 are given, we can calculate matrix C, that is, angular accelerations. Furthermore, by using it, angles and angular velocities at time $t_1+\Delta t$ can be approximated. By repeating this procedure, we can derive the shape of a linear object.

4. COMPUTATION OF LINEAR OBJECT SHAPE AND MEASURING EXPERIMENT

In this section, we show a numerical example by using a proposed procedure in the previous section and demonstrate its validity with a measuring experiment.

The following set of basic functions are used in the computation of this example:

$$e_n(s) = \sin\left(\frac{n\pi}{L} s \right) + \frac{(-1)^{n-1} n\pi}{L} s, \ (n=1, \ldots, 5). \tag{12}$$

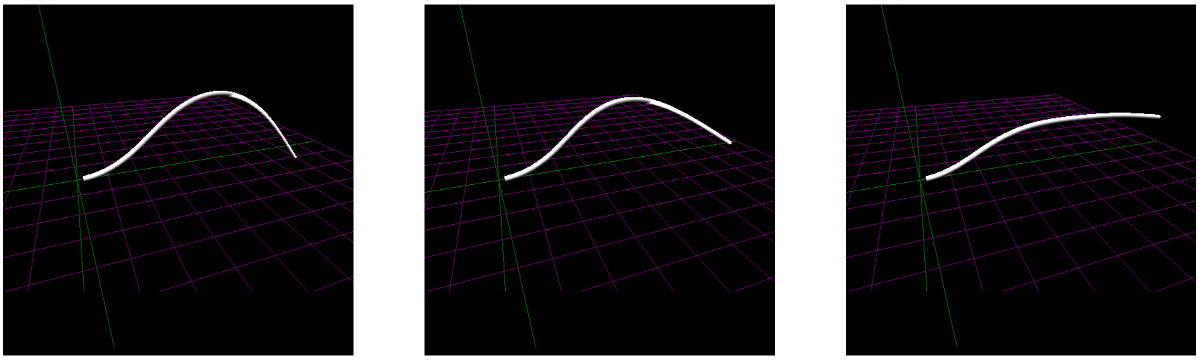
The number of basic functions should be determined with considering both precision in approximation and computing time.

Figure 4 shows an example of computational result. In this example, it is assumed that length, radius, linear density, flexural rigidity, and torsional rigidity of a linear object are 1.0[m], 1.0×10^{-3} [m], 1.0×10^{-3} [kg/m], 1.0×10^{-2} [Nm^2], and 1.0×10^{-1} [Nm^2], respectively. Both endpoints of the object are fixed in the initial state as shown in Figure 4-(a). Geometric constraints are represented as follows:

$$\int_0^L z \,\Big|_{t=0} ds = [0.0 \quad 0.1 \quad 0.8]^T, \ Q(0,0) = [0.0 \quad 0.0 \quad 0.0]^T, \ \frac{\partial Q}{\partial s}(L, 0) = [0.0 \quad 0.0 \quad 0.0]^T. \tag{13}$$

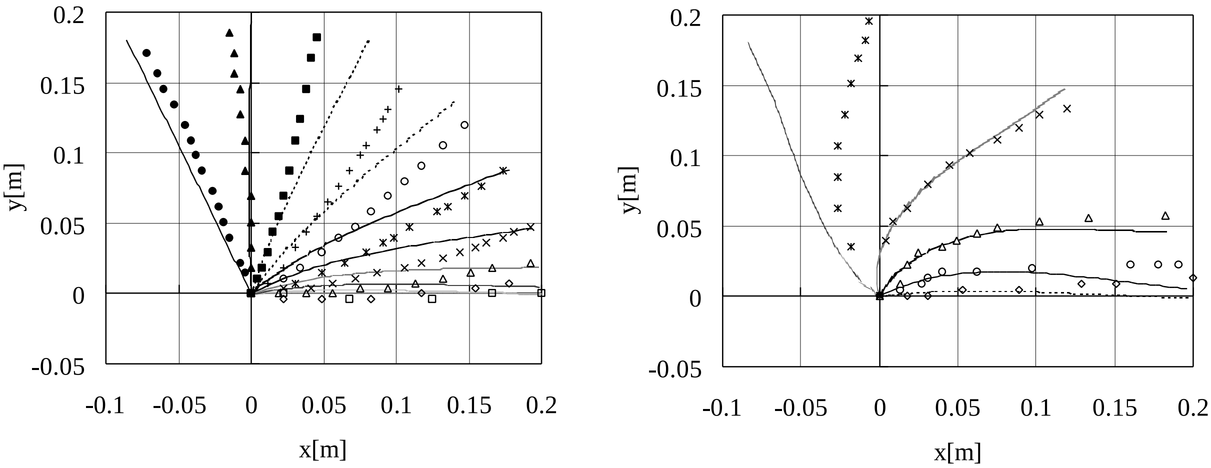
The initial shape of the object can be derived by minimizing potential energy consisting of flexural and torsional energy under constraints given by eq.(13). In detail, see [1]. Figure 4-(b) and (c) illustrates the object shape after its right endpoint is released. Thus, we can computed the shape of a linear object deforming dynamically by using our proposed procedure.

Next, we measure the actual shape of a linear object. A rectangular PET shirt, which is 2.0×10^{-1} [m] long, 1.0×10^{-2}[m] wide, and 1.9×10^{-4}[m] thick, is used as a linear object. Its flexural rigidity is 2.4×10^{-5}[Nm^2] and its linear density is 2.8×10^{-3}[kg/m]. In this measuring experiment, the left endpoint of a linear object is fixed on a manipulator and its right endpoint is free. We measure its shape with a high-speed camera rotating its left endpoint at two kinds of angular acceleration so that its central axis is involved in a horizontal plane through deformation. Computational values and measurement values of the object are plotted in Figure 5. Solid and dotted lines represent computed shapes and measured shapes, respectively. In Figure 5-(a), the left endpoint of the object is rotated at the angular acceleration of 0.4[rad/s^2] and its shapes are illustrated every 320[ms]. In Figure 5-(b), the angular acceleration at P(0,t) is 30[rad/s^2] and results are shown every 80[ms]. The object hardly deforms and computed shapes correspond to measured shapes in Figure 5-(a). In Figure 5-(b), The difference between computed shapes and measured shapes becomes larger as the angular velocity



(a) Initial state (b) 0.10[s] after (c) 0.20[s] after

Figure 4 Computed shapes of linear object deforming dynamically



(a) $\partial^2\theta/\partial t^2(0,t) = 0.4$ [rad/s^2] (b) $\partial^2\theta/\partial t^2(0,t) = 30$ [rad/s^2]

Figure 5 Comparison between computed values and measured values

increases. It seems this difference results from air resistance. It is not considered in our formulation, however, it can not be negligible in this experiment because the width of the PET shirt is not sufficiently small. Therefore, it is expected that we can predict dynamic deformation of a linear object by using our proposed method when its moving velocity is not so high and/or it is very fine.

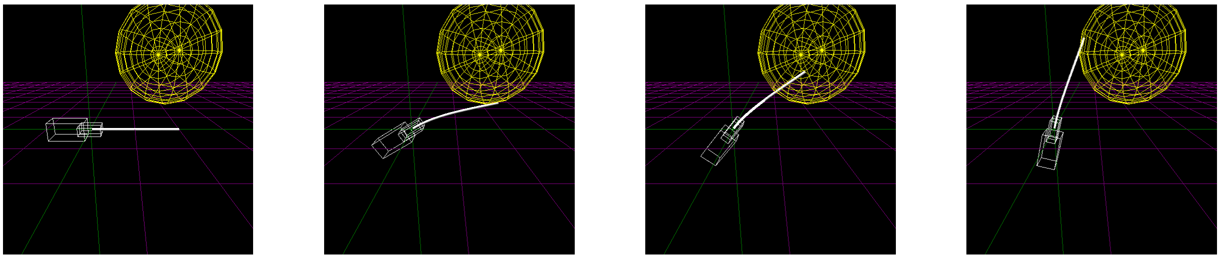
5. APPLICATION TO MANIPULATION PLANNING

In this section, we try applying our proposed method to manipulation planning utilizing dynamic deformation.

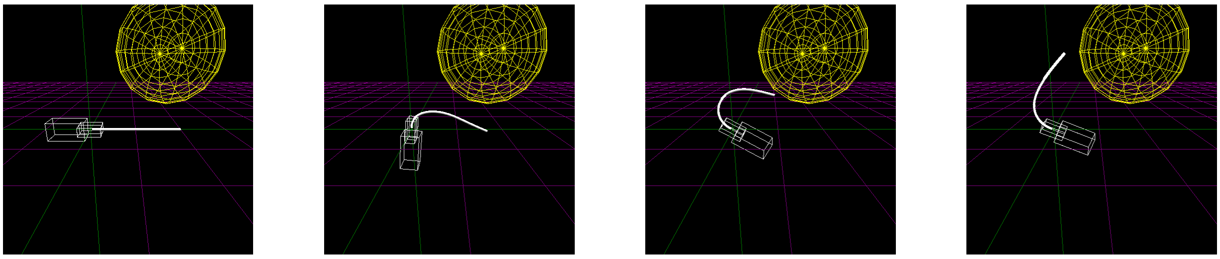
Figure 6 shows an example of avoidance of a spheral obstacle by dynamic deformation. In Figure 6-(a), the left endpoint of a linear object is rotated slowly. Then, it interferes with the obstacle, namely, it collides against the obstacle. In Figure 6-(b), the object is brandished quickly and it can deform without collision against the obstacle. Figure 7 shows trajectories of the right endpoint of a linear object, whose length is 0.2[m] and whose linear density is 3.0×10^{-3}[kg/m], in the following four cases:

- case1 : $R_f = 2.0\times10^{-5}$[Nm^2], $\partial^2\theta/\partial t^2(0,t) = 30$ [rad/s^2],
- case2 : $R_f = 0.4\times10^{-5}$[Nm^2], $\partial^2\theta/\partial t^2(0,t) = 30$ [rad/s^2],
- case3 : $R_f = 0.1\times10^{-5}$[Nm^2], $\partial^2\theta/\partial t^2(0,t) = 30$ [rad/s^2],
- case4 : $R_f = 0.4\times10^{-5}$[Nm^2], $\partial^2\theta/\partial t^2(0,t) = 0.4$ [rad/s^2].

As shown in this figure, the right endpoint passes more inside of a circular orbit as its flexural rigidity decreases or as the angular acceleration increases. By computing this trajectory, we can



(a) Slow manipulation



(b) Quick manipulation

Figure 6 Simulation of manipulation utilizing dynamic deformation

estimate the size of an obstacle which the object can avoid collision against. Therefore, we can determine the angular acceleration of the left endpoint of the object considering the size of the obstacle to avoid. This example is very simple but it seems that we can plan manipulation linear objects utilizing their dynamic deformation by using our proposed method.

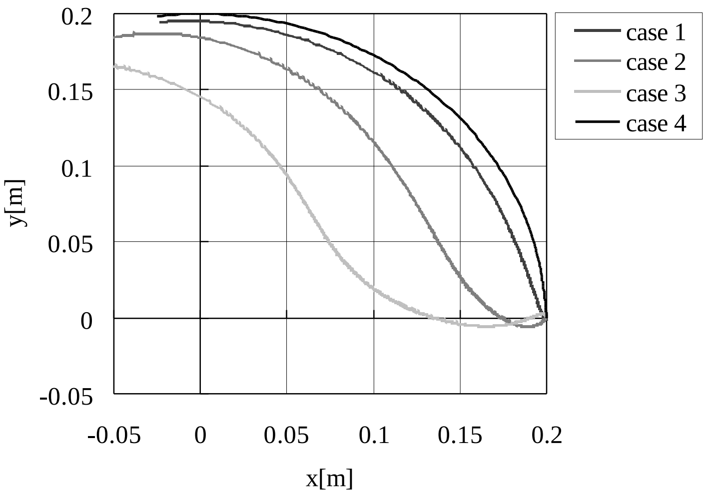


Figure 7 Trajectories of object right endpoint

6. CONCLUSIONS

In this paper, we analyzed dynamic deformation of linear objects. Firstly, a geometric representation of a linear object deforming dynamically was established. It is found that motion and deformation of a linear object can be described by Eulerian angles. Furthermore, its potential energy, its kinetic energy, and geometric constraints imposed on it were formulated by using Eulerian angles. The object shape can be derived by minimizing the time integral of the difference between kinetic energy and potential energy under geometric constraints. Secondly, a procedure to compute the shape was developed and a computational result was shown in order to demonstrate the effectiveness of our proposed procedure. Thirdly, it was verified that our proposed method is appropriate to a dynamic deformation modeling of a linear object by a measuring experiment. Finally, manipulation utilizing dynamic deformation of a linear object can be planed by applying this dynamic analysis of the object. It is expected that our approach will be useful for derivation of task strategies in linear object manipulation.

REFERENCE

[1] Wakamatsu, H., Hirai, S., and Iwata, K., *Modeling of Linear Objects Considering Bend, Twist, and Extensional Deformation*, IEEE Int. Conf. Robotics and Automation, pp.433-438, 1995.